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APPROXIMATE VLF/LF WAVEGUIDE MODE CONVERSION MODEL
Computer applications: FASTMC and BUMP

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JA Ferguson
FP Snyder

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Omega Navigation System Operations Detail

NAVAL OCEAN SYSTEMS CENTER
SAN DIEGO, CALIFORNIA 92152

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> This document reviews vlf radio propagation theory, discusses transequatorial vlf propagation at night and during day-night transitions, and presents a vlf propagation model that includes consideration of inhomogeneities both in the vertical direction and along the propagation path.

The model considers anisotropic effects (caused by the geomagnetic field). It is implemented into a FORTRAN computer program called FASTMC. This program allows for an arbitrarily oriented and elevated transmitting dipole antenna from which the vertical or horizontal electric field at an arbitrary elevation can be output.

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20. Abstract (Cont)

BUMP, a program incorporating the FASTMC capabilities, allows for relatively easy modeling of a moving perturbation with a minimum of data preparation. The FASTMC-BUMP combination is designed specifically for consideration of transequatorial vlf propagation.

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1. VLF RADIO WAVE PROPAGATION TO GREAT DISTANCES

SCOPE OF THIS DOCUMENT

The Omega Navigation System Operations Detail (ONSOD) of the US Coast Guard is responsible for the implementation and evaluation of the Omega Navigation System. Omega uses very low frequency (VLF) radio waves with range from 10.2 to 13.6 kHz and is an operational global system. Thus, a complete evaluation of the system requires an understanding of VLF radio wave propagation under both spatial and temporal operating conditions. A particular set of conditions has been of considerable interest, namely transequatorial propagation at night or during day-night transitions. Theoretical evaluation of such propagation paths requires highly detailed tracking of the propagating modes through the region of the geomagnetic equator because the modes can be very sensitive to the magnetic field. An even more difficult problem is the analysis of the effect of a moving day-night terminator on such paths.

A general background review of VLF radio wave propagation from a theoretical viewpoint is presented, and this is followed by a discussion of transequatorial VLF propagation at night and during day-night transitions. A VLF propagation model which includes consideration of inhomogeneities both in the vertical direction and along the propagation path is then presented. The model includes consideration of the anisotropic effects (caused by the geomagnetic field). It has been implemented into a FORTRAN computer program called FASTMC. This program allows for an arbitrarily oriented and elevated transmitting dipole antenna from which the vertical or horizontal electric field at an arbitrary elevation can be output. An application of the FASTMC capabilities has been further implemented into a program called BUMP, which allows for relatively easy modeling of a moving perturbation with a minimum of data preparation. The FASTMC-BUMP combination is designed specifically for consideration of transequatorial VLF propagation. A discussion of both FASTMC and BUMP is included.

THEORETICAL DEVELOPMENTS

VLF propagation to great distances can be conveniently represented in terms of waveguide mode propagation, where the finitely conducting curved earth and the anisotropic, imperfectly conducting curved ionosphere with dip-

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ping magnetic field form the boundaries of a waveguide. The basic idea that
the earth and lower ionosphere form a waveguide actually goes back to the work
of Watson (1918, 1919), who postulated that the Kennelly-Heaviside layer could
be represented as a conducting shell concentric with the spherical conducting
earth. Watson developed an exact, slowly convergent, harmonic series-type
solution to the EM propagation problem and further devised a transformation of
the series to a highly convergent form. The transformation, which involves
distortion of the contour of integration in the complex wave number plane, is
now identified with Watson's name, although the transformed series is often
"rediscovered" by subsequent workers. Although "waveguides" were not really
"discovered" until the 1930s, the terms in Watson's transformed series were
simply waveguide modes.

The theory of VLF waveguide mode propagation has been presented in the
literature in two different formulations. One formulation, based on the
pioneering work of Watson (1919) and more recently in the theory given by
Schumann (1952, 1954) and developed extensively by Wait (1962), treats the
problem in spherical coordinates wherein the fields are expanded in terms of
azimuthal, longitudinal, and radial functions. In a sequence of simplifying
approximations, including the assumption of azimuthal and longitudinal inde-
pendence of both ground and ionospheric properties, Wait (1962) develops a
modal (eigenvalue) equation in terms of Airy functions. The Airy functions
come about as a result of a third-order approximation to spherical wave
functions (Watson, 1944). The solution of the eigenvalue equation yields the
propagation characteristics of the VLF modes in the earth-ionosphere wave-
guide, and consideration of orthogonality properties of the radial (Airy)
functions yields excitation factors of the modes. The excitation factor is a
quantity that gives the amplitude of the wave excited in a given mode by a
given source. Another simplifying approximation often made involves the
replacement of the inhomogeneous ionosphere with an impedance boundary placed
at the height where the bulk of the VLF energy is assumed to be reflected. A
similar impedance boundary replacement is also made for the earth. The impe-
dance boundary formulation permits specification of the ratio of an electric
field component to a magnetic field component at the boundary without further
consideration of the region beyond the boundary. Analytically, formulation in
terms of impedance boundaries is equivalent to assumption of "homogeneous
boundary conditions" (Friedman, 1959; Morse and Feshbach, 1953). The replace-

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ment of an inhomogeneous, isotropic ionosphere by an "equivalent" impedance boundary is a straightforward procedure. Such a replacement for an anisotropic ionosphere is greatly complicated by the coupling of transverse magnetic and transverse electric polarizations. Boundary conditions must then be formulated in terms of an impedance matrix. Unfortunately, the complication is sufficient to render many of Wait's results inapplicable to the highly anisotropic nighttime ionosphere and to limit their application to the daytime ionosphere, for which the effect of anisotropy is slight. The theory has been significantly modified and extended to anisotropic ionospheres by Galejs (1972).

Another formulation of the earth-ionosphere waveguide problem has been presented by Budden (1961a). In this formulation, a modal equation is developed in terms of reflection coefficients of the ionosphere and the earth. Formulation in terms of reflection coefficients permits determination of propagation characteristics of waveguide modes for essentially any ionospheric or ground conditions, limited only by the ability to compute reflection coefficients for the environment considered. Budden's formulation is essentially planar, although he does point out how to include earth curvature in the direction of propagation in an approximate way by modifying the refractive index in the space between the earth and the ionosphere (Budden, 1962). A more rigorous technique was introduced by Richter (1966) and has been employed in VLF propagation studies by Pappert (1968) and Abbas et al (1971). This technique assumes cylindrical stratification and effects a conformal transformation from cylindrical to Cartesian coordinates. Modal excitation values are determined in Budden's formulation by applying of residue theory rather than by using the modal orthogonality properties of conventional waveguide theory employed by Wait.

Whichever formulation is used at the outset, the use of waveguide mode theory leads to specification of the VLF field as a summation of terms called modes. Because most VLF transmitters in common use radiate a vertically polarized field, only the radial or vertical component of the electric field is usually considered. For a time harmonic source, the VLF modesum for the vertical electric field may be written as

$$E_v(d) = \frac{K(P,f)}{\sqrt{(\sin\ d/a)}}\ \sum_n\ \Lambda_n\ G_n^T\ G_n^R\ e^{-ik_o S_n d}\ , \tag{1}$$

where $K(P,f)$ is a complex constant dependent on transmitted power (P) and frequency (f), d is the distance from the transmitter on a homogeneous, smooth earth of radius a, Λ_n is the excitation factor for mode n, normalized to unity for flat earth, perfectly conducting boundaries, k_o is the free space wave number, S_n is the propagation factor, and $G_n^{T,R}$ represents height-gain functions for mode n, normalized to unity at the ground. One height-gain function is needed for the transmitter (T) and one for the receiver (R). Generally, both Λ_n and S_n are complex. The real part of S_n determines the distance dependence of the phase for a mode while the imaginary part of S_n determines the attenuation rate. Thus

$$\begin{aligned} \alpha_n &= -k_o \text{Im}(S_n) \\ v_n/c &= 1/\text{Re}(S_n), \end{aligned} \tag{2}$$

where α_n is the modal attenuation rate, v_n is the phase velocity, and c is the speed of light in free space.

Numerous assumptions and approximations, some of which will be removed in following sections, are implicit in equation (1). It is assumed that the propagation environment is homogeneous in all but the vertical direction. Thus, the propagation factor, S_n, is independent of position. The influence of the curvature of the earth transverse to the propagation direction is approximately accounted for in the spreading term, sin(d/a). The energy flux for a wave traveling outwards with cylindrical symmetry on a flat earth without attenuation is proportional to 1/r. For a spherically curved earth this energy flux would be proportional to 1/sin(r/a). Thus the usual $1/\sqrt{r}$ term for a cylindrical wave is replaced by the $1/\sqrt{\sin(r/a)}$ term. A careful derivation of the mode equation in spherical coordinates, such as that presented by Wait (1962) and discussed previously, yields the sin(d/a) term as a consequence of an asymptotic approximation to a Legendre function. The mode sum in equation (1) further assumes that "round the world" signals are absent, an approxima-

tion not valid when receiver locations are near the antipode of the transmitter — multipath focusing then occurs.

Extensive results of numerical calculations of VLF waveguide mode constants have been presented in the literature. Early results given by Wait and Spies (1964) show some general properties of VLF modes for typical daytime and nighttime ionospheres, with various ground conductivities and for east-to-west or west-to-east propagation directions at the magnetic equator or when the earth's magnetic field is ignored. In general, the attenuation rate of the lowest order mode is less at night than during the day, and is less for west-to-east propagation than for east-to-west. The attenuation rate increases as ground conductivity decreases — rapidly at low conductivities such as occur for the polar icecaps. Each successive mode order suffers a greater attenuation rate than the previous order mode for both day and night.

Typically, daytime attenuation rates for the first order mode range from 2 to 4 dB/1000 km over the VLF band for sea water ground conditions (essentially equivalent to a perfect conductor at VLF). The nighttime attenuation rates range from about 0.5 to 2 dB/1000 km over the VLF band for the same ground conductivities.

The results of Wait and Spies show that the modal phase velocities for the curved earth can be less than the free space speed of light at intermediate to higher VLF frequencies but increase markedly at lower frequencies. Moreover, phase velocities increase at low ionosphere heights (daytime) compared to higher heights (night), and a decrease in ground conductivity produces a decrease in phase velocity. An increase in phase velocity with mode number is also indicated.

The modal excitation factors and height gains given by Wait and Spies show little dependence on the terrestrial magnetic field. At higher frequencies for isotropic conditions, the excitation factor for the first mode is significantly reduced in comparison with that for a flat earth case. The second mode exhibits a much smaller reduction at higher frequencies. Generally, lower ground conductivities produce an increase in excitation factor. The excitation factor for the second order mode is significantly higher than that for the first order mode, especially at higher frequencies and at night. At night the second order mode may be dominant at the ground to distances of several thousand kilometers. For the first mode the effect of height-gain is

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to cause the dominance of the second mode to be greatly diminished or even reversed. This height-gain increase with height for the first order mode has been called the whispering gallery effect by Budden and Martin (1962) or the earth-detached mode by Wait and Spies (1963). It involves successive reflections from the ionosphere with essentially no intermediate bounce at the ground.

The modal constants characterized above become considerably more complicated with full consideration of the geomagnetic field. From the earliest days of VLF, experimental investigation provided evidence of an apparent violation of the reciprocity principle. Round et al (1925), for example, found that VLF radio waves apparently suffered significantly larger attenuation for propagation in a generally easterly direction than in the reverse direction. Some very early results presented by Wait (1961) for sharply bounded model ionospheres suggested that simple harmonic functions may adequately describe the azimuthal dependence of mode constants under daytime conditions. Further, a numerical modeling study by Ferguson (1968) showed that for daytime conditions, only the horizontal component of the geomagnetic field which is transverse to the direction of propagation is important. This component of the magnetic field is the source of nonreciprocity, and the remaining components were commonly assumed only to alter the propagation characteristics somewhat (Makorov et al, 1970). The most severe geomagnetic field effects were thus expected in the equatorial region for propagation across the magnetic meridians.

Perhaps the first indications of extreme complexity in modal behavior due to the geomagnetic field are found in the numerical modeling results of Snyder (1968a, b) and Pappert (1968). Snyder (1968a) addressed the problem of mode numbering for typical nighttime ionospheres as a function of azimuthal variation for a horizontal magnetic field. Results indicated that a mode with lowest west-to-east phase velocity evolved as a result of continuous variation in magnetic azimuth into a mode in the reverse direction with only second lowest phase velocity. Thus, numbering modes in terms of increasing phase velocity, as is commonly done, results in the evolution of the first order west-to-east mode into the second order mode in the opposite direction, and vice versa. Further analysis (Snyder, 1968b) indicated that for a certain inclination of a dipping geomagnetic field and at a specific (near north-south) azimuth, the two modes considered above became degenerate, forming a single mode. For inclinations more nearly horizontal than the degenerate

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conditions, mode numbering inconsistencies resulted; but for more nearly vertical inclinations, inconsistencies in mode numbering vanished. Pappert (1968) presented the results of numerical modeling for easterly propagation at midlatitudes. Assuming a vertical dipole exciter, Pappert found that, at the high frequency end of the VLF band (only near 30 kHz), modes which are principally transverse electric (TE) may be of importance in a mode sum. Note that a TE mode can be excited by a vertical dipole only because of the presence of the geomagnetic field.

Snyder and Pappert (1969) extended the analysis to consider both easterly and westerly midlatitude nighttime propagation throughout the VLF band as well as to include azimuthal dependencies of mode parameters for a central VLF frequency at both middle and equatorial latitudes. It was found that the importance of principally TE modes is much more pronounced for westerly propagation at midlatitudes than for easterly and that the influence of the principally TE modes on the mode sum can be significant at frequencies as low as 20 kHz or less for westerly propagation. Azimuthal anomalies included drastic polarization changes in going from easterly to westerly paths. For transverse propagation at the magnetic equator, it was shown that modes which are pure transverse magnetic (TM) for propagation to the east may be pure TE for propagation to the west. This is tantamount to the statement that modes which have dominant excitation for easterly propagation may have vanishingly small excitation for westerly propagation. Azimuthal dependencies are characterized by rapid variation of the mode constants in the neighborhood of north-south or south-north propagation, for both equatorial and middle latitudes. These variations manifest themselves in marked differences in mode sums for azimuthal changes as small as 10° or less. Further, it was shown that the maximum total signal attenuation occurs for north-south propagation rather than for east-west propagation.

Although Snyder and Pappert (1969) used hypothetical ionospheres, their results have been further substantiated by Foley et al (1973), who used "realistic models of the ionosphere." Foley et al essentially duplicated the analysis of Synder and Pappert but used ionospheric models presented in the literature by Deeks (1966) and Smith et al (1968). Foley et al concluded that mode characteristics are dependent on the ionosphere model assumed. For example, results with the Smith profile were similar to the results of Snyder and Pappert, whereas under certain circumstances results differed with the Deeks profile. For propagation in the north-south direction with the Deeks

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profile, anomalies were found in the behavior of modes 1 and 2. It would appear that between 16 and 20 kHz, modes 1 and 2 interchange their identities. Although unknown to Foley et al, this identity interchange would seem to be just another manifestation of the modal degeneration reported by Snyder (1968b).

THE TERMINATOR

A manifestation of multimode propagation is found in the phenomenon of sunrise and sunset fading, which is characterized by periodic and repeatable variations in amplitude and phase as the dawn-dusk terminator moves along a VLF propagation path. Crombie (1964) explained the phenomenon using waveguide mode concepts. Of all possible combinations of conditions — sunrise and sunset, position of terminator on propagation path, relative positions of transmitter and receiver, etc — only two fundamental situations occur within the confines of Crombie's explanation.

The first situation depicts the field incident on the terminator from the transmitter as determined by two modes, and the field at the receiver as determined by only one mode. This situation would be characteristic of a transmitter on the night side of the terminator and the receiver on the day side sufficiently removed to avoid multimode propagation. The received signal then depends on the amplitude and phase relationships of the modes incident on the terminator and on the efficiency with which these modes are converted by the terminator to the single mode which reaches the receiver. Because the modal relationships are determined by the distance of the terminator from the transmitter, periodic variations will occur in the received signal as the terminator moves along the propagation path. The periods will be determined by the difference in the phase velocities of the two modes entering the terminator, and the variations must occur simultaneously for all points in the portion of the propagation path beyond the terminator region. The fading would be most intense when the two entering modes have the most pronounced interference — usually when the terminator is near the transmitter.

The second situation depicts the incident field as a single mode. The presence of the terminator leads to multimode generation in the region past the terminator, which in turn leads to modal interference. In this situation, the interference pattern would appear to be attached to the terminator and to move synchronously with it. Signal minima are then to be expected at fixed distances from the terminator, the distance between fades being determined by the difference between the phase velocities of the two modes leaving the terminator region. The deepest signal fades would be expected as the terminator approaches the receiver. This situation corresponds to propagation from the sunlit side of the terminator into the night side. Note that in both

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situations, the fade spacing would be determined by the differences between the phase velocities of two nighttime modes.

A thorough experimental examination of the Crombie model was conducted by Walker (1965), who recorded 18 kHz transmissions from NBA Balboa, Panama, while making repeated shipboard crossings of the Atlantic in the equatorial region. During sunrise (propagation from night to day) signal fades were observed simultaneously at all points in the day portion of the path — in agreement with the first situation discussed above. Walker determined the change in terminator position between successive fades and from this calculated the required differences in the phase velocities of the two interfering modes. These results were in good agreement with theoretical results for nighttime ionospheres, further substantiating the Crombie model. Observations were also made during sunset (propagation from day to night) and results were in excellent agreement with the Crombie model.

EQUATORIAL ANOMALY

The observations just discussed were, for the most part, made at middle or low latitudes. As the previous discussion indicates, such observations have been explained adequately by the use of quite simple VLF waveguide mode theory. However, some observations of VLF transmissions over long transequatorial paths have apparently indicated an anomalous effect associated with nighttime or transitional transequatorial VLF propagation.

Of the reports on transequatorial VLF propagation, one of the earliest to show an apparent anomaly was by Chilton et al (1964). They made simultaneous observations of NBA transmissions (Canal Zone, Panama) at 18 kHz for paths to both the northern and southern hemispheres. Both paths were of similar length, but the southern hemisphere path crossed the magnetic equator. Although they did not look at transition fading, they did observe an anomalous difference in the diurnal changes of signal amplitude and phase. Chilton et al suggested that their observations resulted from a difference in ionization profile due to latitudinal dependence of cosmic rays. More recently, Chilton and Crary (1971) suggested that the latitude-dependent ionization source might arise from the recently discovered x-ray stars.

Araki et al (1969) recorded dual frequency (15.5 and 22.3 kHz) VLF signals in Japan from station NWC in Australia. They reported sunrise fading on 22.3 kHz but not on 15.5 kHz and further noticed that during fading at 22.3 kHz the first phase change was an increase followed by a phase decrease. The nighttime field strength was somewhat less at 15.5 kHz and somewhat higher at 22.3 kHz than during the day. Araki et al (1969) concluded that their results suggest the existence of a frequency-dependent anomaly of the nighttime field relative to the daytime field along their transequatorial propagation path.

Bickel et al (1970) conducted an experiment to investigate the apparently anomalous results of Snyder and Pappert (1969), who predicted maximum signal attenuation in a southerly rather than westerly direction. Measurements of 23.4 kHz signals from station NPM (Lualualei, Hawaii) were made aboard an airplane as it flew radials along the propagation paths to Seattle, Ontario (California), Samoa, and Wake Island. Bickel et al confirmed the Snyder and Pappert (1969) results and concluded that the increased southerly attenuation is an effect of the geomagnetic field. An unexplained effect, however, was observed. For the radial to Samoa, rapid signal amplitude variations with

distance were observed in the vicinity of the geomagnetic equator. Although equipment problems may be suspected, other Hawaii-based transmissions simultaneously recorded by means of completely separate equipments displayed similar results (Bickel, private communication, 1970).

The first report of an equatorial anomaly associated with transition fading was made by Lynn (1967). He reported on VLF transmissions at 18.6 kHz from NLK (Jim Creek, Washington) to Smithfield, South Australia. This propagation path is approximately 13 500 km long, and the receiver-to-geomagnetic-equator distance along the path is approximately 5700 km. The direction of propagation is essentially southwesterly. The interference distance reported by Lynn for sunrise transition fading is approximately 2000 km for the terminator located in midlatitudes (in excess of $\pm 20°$ from the geomagnetic equator). Whenever sunrise transition fading was observed while the terminator was within $\pm 20°$ of the geomagnetic equator, the interference distance increased to as much as 3700 km; and when averaged over the anomaly, it had a value of 2900 km. Lynn (1967) concluded from his observations that the change in interference distance resulted from a change in the difference of phase velocity for two modes as well as from a change in the relative phases of the appropriate mode conversion coefficients. He left unanswered the question of a possible cause for these changes. It is interesting to note that Lynn's conclusions are dependent on his selection of both a VLF propagation model and a mode conversion model. Lynn (1967) assumed an isotropic propagation model wherein any changes in propagation parameters necessitates changes in ionospheric conditions.

Kaiser (1968) examined VLF signals at 18.6 and 24 kHz from NLK and at 20 kHz from WWVL, received at Lower Hutt, New Zealand. These propagation paths are similar to the path considered by Lynn (1967), and Kaiser's results are much the same as Lynn's. An anomalously large interference distance (in excess of 3000 km) was observed for sunrise transition fading when the terminator was within about $\pm 20°$ of the magnetic equator. Kaiser also examined transition fading data previously published elsewhere and concluded that anomalously large values of the sunrise interference distance are characteristic of generally east-to-west VLF propagation below about 30° magnetic latitude. A further conclusion by Kaiser (1968) is that any anomaly in sunset transition fading for such paths is relatively minor. Kaiser suggests that a

possible cause of the anomaly might be a larger and sharper day-night wave-guide transition near the geomagnetic equator as compared to higher latitudes. He does not suggest, however, a possible cause for the latitudinal dependence of the day-night waveguide transition.

Additional observations of this transition fading anomaly have been reported by Lynn (1969, 1970). These observations further substantiate the general characteristics of the anomaly. A theoretical interpretation of the transequatorial anomaly was presented by Lynn (1970). His interpretation is based only on assumed latitudinal variations in modal phase velocities and relative phases of the mode conversion coefficients. He does not suggest a possible cause for the variations.

Meara (1973) has extended Lynn's (1970) analysis to include determination of the changes in phase velocities within the equatorial anomaly region. He concludes that the phase velocity of the first mode is essentially unaltered by propagation through the equatorial region, whereas that of the second mode is reduced. Note that Meara's results are entirely dependent on the assumed propagation and mode conversion models. Once again, no explanation of a possible cause for the variations in phase velocities is given.

A further application of Lynn's (1970) analysis to explain some anomalous diurnal changes in transequatorial VLF propagation has been made by Araki (1973). The analysis follows essentially that previously discussed. Araki does go a step further than Lynn (1970) and Meara (1973), however, in that he assumes the validity of the isotropic propagation model and asserts that any change in propagation parameters results from a modification of the nighttime equatorial ionosphere. Lynn (1978) has pointed out that the propagation paths of Chilton et al (1964) and Araki (1973) have a west-east component rather than an east-west component as have the other paths discussed here. He further claims that only a minor azimuthal variation in relative phase velocities of interfering nighttime modes can account for the observations of Chilton et al (1964) and Araki (1973). Lynn (1978) thus concludes that the observations of Chilton and Crary (1971) cannot be seen as direct evidence for the control of night VLF reflection heights by stellar X-ray sources. But Svennesson and Westerlund (1979), using waveguide mode theory combined with nighttime profiles given by ionospheric theory, claim results that support the reports of X-ray star effects on VLF radio signals.

To address the question of transequatorial VLF propagation at night or during transition periods requires theoretical VLF radio wave propagation modeling that includes consideration of inhomogeneities along the propagation path as well as in the vertical direction. Anisotropic effects due to the geomagnetic field must also be considered. A propagation model with these capabilities has been developed and is discussed in the following sections.

2. WAVEGUIDE MODE THEORY — HORIZONTALLY HOMOGENEOUS GUIDES

COORDINATE TRANSFORMATION

A review of the waveguide mode theory for VLF propagation in the earth-ionosphere waveguide, homogeneous in and transverse to the direction of propagation, is presented in this section. The earth-ionosphere guide is modeled as cylindrical, with effects of curvature transverse to the direction of propagation accounted for through the use of an appropriate "spreading term" as described in Section 1. Also, as mentioned there, the formulation employed in this study is based on the pioneering work of Budden (1961, 1962). Because Budden's formulation is essentially planar, a coordinate transformation is employed to convert between cylindrical geometry and planar geometry. The transformation, introduced by Richter (1966) with respect to earth flattening techniques and used by Pappert (1968) and Abbas et al (1971) in VLF propagation studies, is briefly outlined.

Application of the conformal coordinate transformation of the form $\rho = ae^{z/a}$; $\phi = x/a$ results in the conversion of circles (ρ = constant) in the ρ,ϕ plane to straight lines (z = constant) in the x,z plane. If $\underline{a}$ represents the radius of the earth, the transformation maps the surface of the earth ($\rho = a$) to the line $z = 0$. The transformation further maps radii (ϕ = constant) from the ρ,ϕ plane to lines x = constant in the x,z plane. A full circular arc ($\phi = 2\pi$) at $\rho = 0$ is mapped so that the length of x is the circumference of the earth.

The transformation used here differs from that employed by Richter (1966), Pappert (1968), and Abbas et al (1971) in that they maintained some selected altitude (h) as invariant in both coordinate frames. However, the two transformations differ only in quantities of second order in smallness. As the invariant altitude (h) approaches zero, the two transformations become identical.

Maxwell's curl equations with time variation of the form $e^{i\omega t}$ for a medium characterized by a dielectric tensor may be written as

$$\underline{\nabla} \times \underline{E} = -ik_o\underline{H}$$
$$\underline{\nabla} \times \underline{H} = ik_o\underline{\underline{\epsilon}} \cdot \underline{E}, \tag{3}$$

where k_o is the wave number of free space ($k_o = \omega/c$) and $\underset{\approx}{\epsilon}$ is the dielectric tensor. In equations (3), the variable $\underset{\sim}{H}$ is actually the magnetic intensity multiplied by the free space impedance. Assuming $\partial/\partial y = 0$, equation (3) in cylindrical coordinates may be written

$$\frac{1}{\rho} \frac{\partial E_y}{\partial \phi} = -ik_o H_\rho, \qquad \frac{1}{\rho} \frac{\partial H_y}{\partial \phi} = ik_o (\underset{\approx}{\epsilon} \cdot \underset{\sim}{E})_\rho$$

$$\frac{\partial E_y}{\partial \rho} = ik_o H_\phi, \qquad \frac{\partial H_y}{\partial \rho} = -ik_o (\underset{\approx}{\epsilon} \cdot \underset{\sim}{E})_\phi \qquad (4)$$

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho E_\phi) - \frac{1}{\rho} \frac{\partial E_\rho}{\partial \phi} = -ik_o H_y, \qquad \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho H_\phi) - \frac{1}{\rho} \frac{\partial}{\partial \rho} H_\rho = ik_o (\underset{\approx}{\epsilon} \cdot \underset{\sim}{E})_y \; .$$

Applying the coordinate transformation to equation (4):

$$\frac{\partial E_y}{\partial x} = -ik_o \frac{\rho}{a} H_\rho, \qquad \frac{\partial H_y}{\partial x} = ik_o \frac{\rho}{a} (\underset{\approx}{\epsilon} \cdot \underset{\sim}{E})_\rho \qquad (5a)$$

$$\frac{\partial E_y}{\partial z} = ik_o \frac{\rho}{a} H_\phi, \qquad \frac{\partial H_y}{\partial z} = -ik_o \frac{\rho}{a} (\underset{\approx}{\epsilon} \cdot \underset{\sim}{E})_\phi \qquad (5b)$$

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$$\frac{\partial}{\partial z} \frac{\rho}{a} H_\phi - \frac{\partial}{\partial x} \frac{\rho}{a} H_\rho = ik_o \frac{\rho^2}{a^2} (\underset{\approx}{\epsilon} \cdot \underset{\approx}{E})_y;$$

(5c)

$$\frac{\partial}{\partial z} \frac{\rho}{a} E_\phi - \frac{\partial}{\partial x} \frac{\rho}{a} E_\rho = -ik_o \frac{\rho^2}{a^2} H_y.$$

By making the identifications

$$E_x = \rho/a \; E_\phi; \quad E_y = E_y; \quad E_z = \rho/a \; E_\rho;$$

$$H_x = \rho/a \; H_\phi; \quad H_y = H_y; \quad H_z = \rho/a \; H_\rho,$$ (6)

equations (5) become identical to what would be obtained in a rectangular coordinate frame for a medium characterized by permeability tensor and dielectric tensor of the form

$$\underset{\approx}{\mu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{2z/a} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ (7)

$$\underset{\approx}{\epsilon} = \begin{pmatrix} \epsilon_{\phi\phi} & e^{z/a} \epsilon_{y\phi} & e_{\rho\phi} \\ e^{z/a}\epsilon_{\phi y} & e^{2z/a}\epsilon_{yy} & e^{z/a}\epsilon_{\rho y} \\ \epsilon_{\phi\rho} & e^{z/a}\epsilon_{y\rho} & \epsilon_{\rho\rho} \end{pmatrix}.$$ (8)

Some additional insight is obtained by considering the transformed equations in the free space region between the earth and the ionosphere. Making the usual substitutions into equation (5c) from equations (5a) and

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(5b), along with the free space assumption, yields the equations

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}\right) E_y = -k_o^2 \, e^{2z/a} \, E_y \tag{9a}$$

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}\right) H_y = -k_o^2 \, e^{2z/a} \, H_y. \tag{9b}$$

Thus, the effect of the transformation is to replace the original cylindrical free space region with a planar region filled with a medium which has refractive index equal to $e^{z/a}$ for TE polarized waves or which has permeability equal to $e^{2z/a}$ for TM polarized waves.

MODE EQUATION

The prototype waveguide model considered here consists of two plates of infinite extent, parallel to the x-y plane and located at z = 0 and z = h. The interior of the guide is assumed to be free space. This prototype waveguide corresponds to the "fictitious free space region" to be later discussed in this section, where it is used to determine modal excitation factors and polarization coupling. The electromagnetic field is assumed to be independent of the y-coordinate and propagation is assumed to be in the x-direction. Any given waveguide mode is considered to be composed of two crossing plane waves, upgoing and downgoing, with wave normals at the angles $\pm\theta$ to the x-axis.

The nature of the boundary planes will be specified in terms of reflection coefficients. These coefficients are defined in terms of the ratios of specific components of the upgoing and downgoing waves and are therefore dependent on the polarization of the waves. A convenient way of describing the polarization is in terms of the linear components of the complex electric vector which are perpendicular and parallel to the plane of incidence (the x-z plane). The notation ∥(⊥) will indicate that the electric vector is parallel (perpendicular) to the plane of incidence.

In general, the upper boundary must be considered anisotropic. Thus there can be a change in polarization upon reflection. The reflecting properties of the upper boundary are given by reflection coefficients having the matrix form

$$\underset{\approx}{R} = \begin{pmatrix} {}_{\parallel}R_{\parallel} & {}_{\perp}R_{\parallel} \\ {}_{\parallel}R_{\perp} & {}_{\perp}R_{\perp} \end{pmatrix} , \tag{10}$$

where the first subscript describes the polarization of the incident (upgoing) wave and the second subscript denotes the polarization of the reflected wave. The properties of the lower boundary are assumed isotropic and specified by a diagonal reflection matrix of the form

$$\overline{\underset{\approx}{R}} = \begin{pmatrix} {}_{\parallel}\overline{R}_{\parallel} & 0 \\ 0 & {}_{\perp}\overline{R}_{\perp} \end{pmatrix} . \tag{11}$$

If the upgoing wave is considered as the primary or incident wave, the downgoing wave can be considered as resulting from reflection at the upper boundary. This wave will in turn be reflected at the lower boundary, giving rise to another upgoing wave. The requirement that the second upgoing wave be the same as the original yields the mode condition. This process is shown in the diagram below.
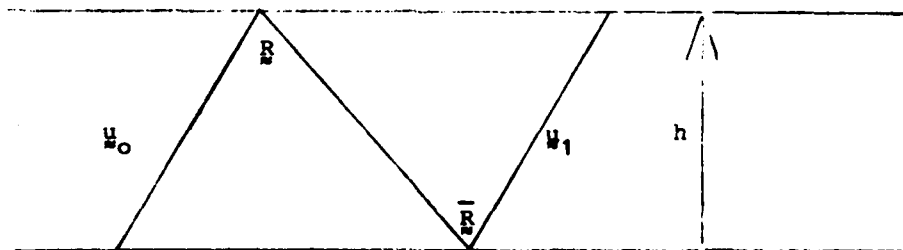
The primary wave is represented by the column matrix $\underset{\approx}{u}_0$. The second upgoing wave ($\underset{\approx}{u}_1$) is therefore given by the relationship

$$\underset{\approx}{u}_1 = \overline{\underset{\approx}{R}} \, \underset{\approx}{R} \, \underset{\approx}{u}_0 \, \exp[-2ikh\sin(\theta)] ,$$

where the exponential term accounts for phase change in the double passage across the guide. The self-consistency requirement for the waveguide mode is thus

$$\det \left\{ \overline{R}\, R\, \exp\left[-2ikh\sin(\theta)\right] - 1 \right\} = 0, \tag{12}$$

where 1 is the unit diagonal matrix.

POINT SOURCES AND LINE SOURCES

The source is assumed to be a vertical Hertzian dipole. Budden (1961, 1962) has shown that a vertical dipole is equivalent to an infinite set of line quadrupoles, where the line quadrupoles are located parallel to the x-y plane and inclined at (complex) angles to the y-axis. This is easily seen by considering the Hertz vector for a dipole at the origin. If the dipole is parallel to the z-axis, the Hertz vector has only a z-component and is proportional to exp(-ikr)/r. For a dipole of unit strength, the Hertz vector is as follows (Budden, 1962):

$$\frac{e^{-ikr}}{4\pi r} = -\frac{k}{8\pi} \int_{-\pi/2-i\infty}^{\pi/2+i\infty} \cos(\alpha)\, H_1^{(2)}(ks)\, d\omega, \tag{13}$$

where

$$\begin{aligned}
\alpha &= \arctan\left\{ z/\left[x\cos(\omega) + y\sin(\omega)\right] \right\} \\
s &= \left\{ \left[x\cos(\omega) + y\sin(\omega)\right]^2 + z^2 \right\}^{1/2}.
\end{aligned} \tag{14}$$

Thus s is the distance from a point (x, y, z) to a line in the x-y plane passing through the origin at an angle ω to the y-axis, and α is the angle between the x-y plane and the line along s. $H_1^{(2)}$ is a Hankel function of the

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second kind, of order one. The term $\cos(\alpha)H_1{}^{(2)}(ks)$ would result from a line quadrupole source in the x-y plane at an angle ω to the y-axis. Consider a line quadrupole source as composed of two line dipole sources at $x = \frac{1}{2}\,\delta x$ and $x = -\frac{1}{2}\,\delta x$ in the x-y plane. The line dipoles consist of closely spaced dipoles oriented parallel or antiparallel to the z-axis and distributed parallel to the y-axis. The Hertz vector for the line dipoles is proportional to $H_o{}^{(2)}(k\rho)$ where $\rho = (x^2 + z^2)^{1/2}$. Letting δx tend to zero and letting the strengths (M, -M) of the line dipoles increase in such a way that the product $M\delta x$ remains finite will result in a term proportional to $\partial H_o{}^{(2)}(k\rho)/\partial x$, which reduces to a term of the form $\cos(\theta)H_1{}^{(2)}(k\rho)$, where $\cos(\theta) = x/\rho$. From the definitions of s and α in (14), it is clear that the integral in (13) results from an infinite set of line quadrupoles in the x-y plane, each at a different angle ω to the y-axis. Because the path of integration in (13) is not unique, there is an infinite number of ways in which the angles ω may be chosen.

Consider a guide bounded by perfectly conducting plates with a line source parallel to the y-axis. This source generates waveguide modes, and each consists of crossing plane waves with normals in the x-y plane at angles $\pm\theta_m$ to the x-axis. For a line source at an angle ω to the y-axis, the component plane wave has the form $\exp\left(-ik\{[x\,\cos(\omega) + y\,\sin(\omega)]C_m + z\,S_m\}\right)$, where $C_m = \cos(\theta_m)$ and $S_m = \sin(\theta_m)$. A point source within the guide is now represented by an infinite set of line sources such that the total field of the component wave is proportional to

$$\int_{-\pi/2-i\infty}^{\pi/2+i\infty} \exp\left(-ik\{[x\,\cos(\omega) + y\,\sin(\omega)]C_m + z\,S_m\}\right)\,d\omega \ . \qquad (15)$$

Making the substitutions $\rho = \left(x^2 + y^2\right)^{1/2}$ and $\cos(u) = x/\rho$ in (15),

$$\int_{-\pi/2-i\infty}^{\pi/2+i\infty} \exp\left(-ik\rho C_m \cos(\omega - u)\right)\,\exp\left(-ikz S_m\right)\,d\omega \ . \qquad (16)$$

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The integration in (16) yields a Hankel function of the second kind, of order zero (Sommerfeld, 1949). Thus the total field of the component wave is proportional to

$$H_o^{(2)}(k\rho C_m) \exp(-ikzS_m).$$

The appropriate Hankel function exchange for the exponential term when a line source is used is therefore

$$\exp(-ikxC_m) \rightarrow H_o^{(2)}(k\rho C_m). \tag{17}$$

MODAL EXCITATION FACTORS AND POLARIZATION COUPLING

The excitation of waveguide modes is conveniently treated in terms of line sources. The technique consists of selecting one of the possible line sources for the integrand in (13), finding the mode excitation for this line source, then effecting a change from an exponential term to the appropriate Hankel function. Following the method of Budden (1961, 1962), the source is a line of quadrupoles at height d (near the ground) parallel to the y-axis. A fictitious free space region is introduced that extends from z = l (lower) to z = u (upper) such that l < d < u (the line source is assumed within the free space region). The region above z = u is characterized by the reflection coefficient matrix $\underset{\sim}{R}_u$, while the region below z = l is characterized by a reflection coefficient matrix $\overline{R}_l$. If the line source has unit strength, the y-component of the magnetic field in infinite free space can be written

$$H_y = \frac{k_o^3}{4\pi\epsilon_o} \int_{0-i\infty}^{\pi+i\infty} \exp(ik_o(xC+|z-d|S))C^2 d\theta , \tag{18}$$

where $C = \cos(\theta)$, $S = \sin(\theta)$, and the limits of integration are selected to ensure convergence for all x (Sommerfeld, 1949). The primary field integrand in equation (18) represents a plane wave which undergoes reflections at z = u and z = l, and these reflected waves must be added to the primary field to obtain the total field. Four combinations are possible, two from waves

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initially upgoing or downgoing at the source and two from waves reaching the reception point upgoing or downgoing.

\n initially upgoing wave is converted to a downgoing wave by the reflection process denoted by the operator $\left(\underline{R}_u\underline{\overline{R}}_1\right)^N\underline{\overline{R}}_u$ and to an upgoing wave by the operator $\left(\underline{\overline{R}}_1\underline{R}_u\right)^N$. An initially downgoing wave is converted to an upgoing wave by the operator $\left(\underline{\overline{R}}_1\underline{R}_u\right)^N\underline{\overline{R}}_1$ and to a downgoing wave by the operator $\left(\underline{R}_u\underline{\overline{R}}_1\right)^N$. The N denotes N reflection processes of the the type described by the bracketed term. With each reflection process must also be associated phase terms to account for the transversals to and fro between z = 1 and z = u. The subscripts u and 1 denote reflection coefficients evaluated at z = u and z = 1, respectively.

The total field is obtained by combining the primary and all reflected fields and letting the dimensions of the free space region tend to zero. This requires taking the limits 1 → u and z → d in such a way that 1 < d < u is maintained. The total field may be written as

$$H_y = \frac{k_o^3}{4\pi\varepsilon_o}\sum_{N=0}^{\infty}\int e^{-ik_o xC}\left(\underline{\overline{R}}\,\underline{R}\right)^N \underline{P}\, C^2 d\theta$$

$$+\frac{k_o^3}{4\pi\varepsilon_o}\sum_{N=0}^{\infty}\int e^{-ik_o xC}\left(\underline{\overline{R}}\,\underline{R}\right)^N \underline{\overline{R}}\, \underline{P}\, C^2 d\theta$$

$$+\frac{k_o^3}{4\pi\varepsilon_o}\sum_{N=1}^{\infty}\int e^{-ik_o xC}\left(\underline{R}\,\underline{\overline{R}}\right)^N \underline{P}\, C^2 d\theta$$

$$+\frac{k_o^3}{4\pi\varepsilon_o}\sum_{N=1}^{\infty}\int e^{-ik_o xC}\left(\underline{R}\,\underline{\overline{R}}\right)^N \underline{R}\, \underline{P}\, C^2 d\theta \ . \tag{19}$$

The reflection coefficients are evaluated at $z = d$ and the subscripts l and u are no longer needed. In equation (19), the first summation starts with $N=0$ in order to include the primary, nonreflected wave; and $\underset{\sim}{P}$ is a column matrix whose second element is zero, corresponding to a vertical electric dipole source.

The order of summation and integration is now reversed so that each integrand will contain a geometric series. It can be shown (Budden, 1961, 1962) that the series converges for certain simplified cases, and convergence is assumed here. Summation of the geometrical series gives

$$\sum_{N=0}^{\infty} \left(\overline{\underset{\sim}{R}} \, \underset{\sim}{R} \right)^{N} = \left(\underset{\sim}{1} - \overline{\underset{\sim}{R}} \, \underset{\sim}{R} \right)^{-1} \tag{20}$$

$$\sum_{N=1}^{\infty} \left(\underset{\sim}{R} \, \overline{\underset{\sim}{R}} \right)^{N} = \left(\underset{\sim}{R} \, \overline{\underset{\sim}{R}} \right) \left[\left(\underset{\sim}{1} - \underset{\sim}{R} \, \overline{\underset{\sim}{R}} \right) \right]^{-1}$$
$$= \underset{\sim}{R} \left(\underset{\sim}{1} - \overline{\underset{\sim}{R}} \, \underset{\sim}{R} \right)^{-1} \overline{\underset{\sim}{R}} \tag{21}$$

$$\sum_{N=1}^{\infty} \left(\underset{\sim}{R} \, \overline{\underset{\sim}{R}} \right)^{N} \underset{\sim}{R} = \left(\overline{\underset{\sim}{R}} \right)^{-1} \sum_{N=1}^{\infty} \left(\overline{\underset{\sim}{R}} \, \underset{\sim}{R} \right)^{N}$$
$$= \underset{\sim}{R} \left(\underset{\sim}{1} - \overline{\underset{\sim}{R}} \, \underset{\sim}{R} \right)^{-1}. \tag{22}$$

Consequently the result of summing the series in equation (19) is

$$H_{y} = \frac{k_{o}^{3}}{4\pi\varepsilon_{o}} \int_{0-i\infty}^{\pi+i\infty} e^{-ik_{o}xC} \left(\underset{\sim}{1} + \underset{\sim}{R} \right) \left(\underset{\sim}{1} - \overline{\underset{\sim}{R}} \, \underset{\sim}{R} \right)^{-1} \left(\underset{\sim}{1} + \overline{\underset{\sim}{R}} \right) \underset{\sim}{P} \, C^{2} d\theta. \tag{23}$$

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If the sign of θ is reversed, the role of incident and reflected wave is exchanged and each reflection element is replaced by its inverse. The integrand simply changes sign upon reversal of the sign of θ. The integral may be evaluated by using the theory of residues, deforming the contour to run symmetrically through the origin from $\pi/2 + i\infty$ to $-\pi/2 - i\infty$. Because the integrand is antisymmetric about $\theta = 0$, the integral along the new contour is zero. In changing the contour, however, singularities are crossed. The most important of these are poles of the factor $\left(\underset{\sim}{1} - \overline{\underset{\sim}{R}}\ \underset{\sim}{R}\right)^{-1}$. The residue contribution of a simple pole at θ_n is as follows:

$$H_{yn} = \frac{ik_o^3}{2\epsilon_o}\ e^{-ik_o x C_n}\ \frac{\left(\underset{\sim}{1} + \underset{\sim}{R}\right)_n\ \underset{\sim}{X}_n \left(1 + \overline{\underset{\sim}{R}}\right)_n\ \underset{\sim}{R}\ C_n^2}{\left(\frac{\partial F}{\partial \theta}\right)_{\theta_n}}, \tag{24}$$

where

$$F = \det\left(\underset{\sim}{1} - \overline{\underset{\sim}{R}}\ \underset{\sim}{R}\right)$$

$$X_n = \lim_{\theta \to \theta_n}\left[F\left(\underset{\sim}{1} - \overline{\underset{\sim}{R}}\ \underset{\sim}{R}\right)^{-1}\right], \tag{25}$$

and the subscript, n, signifies that the quantities are evaluated at $\theta = \theta_n$. The pole condition $F = 0$ is identical to the waveguide mode equation (12) in the limit $h \to 0$. Thus the solution of the pole condition yields the waveguide modes, and the residues at the poles give the excitation factors. Performing the matrix multiplications in equation (24), using the Hankel function substitution [equation (17)], and employing the geometrical spreading factor yields the equation

$$H_{yn} = \frac{-k_o^{5/2}\ C_n^{3/2}\ e^{i\frac{\pi}{4}}}{2\epsilon_o\ \sqrt{2\pi a \sin(\rho/a)}}\ \frac{\left(1 + \ _\parallel\overline{R}_\parallel\right)^2\left(1 - \ _\parallel\overline{R}_{\perp\perp}R_\perp\right)}{_\parallel\overline{R}_\parallel\ \left(\frac{\partial F}{\partial \theta}\right)_{\theta_n}}\ e^{-ik_o \rho C_n}. \tag{26}$$

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To be compatible with the mode sum of Section 1 [equation (1)], it is necessary to change the reference direction for θ from the x-axis to the z-axis. Doing so simply requires interchanging the factors S and $\sin(\theta)$ with the factors C and $\cos(\theta)$.

Equation (26) requires the source and reception point to be at the same height, which is also the height at which the reflection elements are to be determined. If the dependence of H_{yn} on height z is given by the function $G_n(z)$, then an obvious extension of equation (26) to include arbitrary source and reception height involves multiplication by the function

$$\frac{G_n(z_T)}{G_n(d)} \; \frac{G_n(z_R)}{G_n(d)} \; ,$$

where z_T is the transmitter height, z_R is the receiver height, and d is the evaluation height for the reflection elements, assumed to be below the ionosphere.

Whenever the ionosphere is assumed to be isotropic (no geomagnetic field), the field components can be divided into two sets. These are the well-known TE ($H_y = 0$) and TM ($E_y = 0$) modes. When anisotropy is included, however, a given mode, in general, involves both H_y and E_y. To determine the coupling between the TM and TE components of the eigenfunctions, we again introduce a thin fictitious region of free space within the transformed planar waveguide. The region is assumed to be centered at some altitude z = d which is below the ionosphere, so that the region z < d can be considered isotropic. Within this region, the fields of the TM and TE components can be considered in terms of upgoing and downgoing plane waves. Furthermore, the eigenfunctions satisfy the condition that a particular component, say the upgoing, must be returned to its original value after being reflected from the two boundaries of the thin free space region. Thus, for the upgoing component $e_\perp^u$ (the component perpendicular to the plane of propagation), the requirement is that

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$$e_\perp^u = {}_\perp\bar{R}_\perp \; {}_\perp R_\perp \, e_\perp^u + {}_\perp\bar{R}_\perp \; {}_\parallel R_\perp \, e_\parallel^u$$

or

$$\frac{e_\parallel^u}{e_\perp^u} = \frac{1 - {}_\perp\bar{R}_\perp \; {}_\perp R_\perp}{{}_\perp\bar{R}_\perp \; {}_\parallel R_\perp}. \tag{27}$$

In the free space region

$$e_\parallel^u = \cos(\theta)\, e_x^u - \sin(\theta)\, e_z^u \equiv h_y^u$$
$$e_\perp^u = e_y^u,$$

where h_y^u is the upgoing component of H_y and e_y^u is the upgoing component of E_y. The total H_y and E_y fields within the free space gap at $z = d$ are therefore represented by the equations

$$E_y = e^{-ik_o Cd}\left(1 + \frac{1}{{}_\perp\bar{R}_\perp}\right)$$

$$H_y = \left(\frac{1 - {}_\perp\bar{R}_\perp \; {}_\perp R_\perp}{{}_\perp\bar{R}_\perp \; {}_\parallel R_\perp}\right)\left(1 + \frac{1}{{}_\parallel\bar{R}_\parallel}\right)e^{-ik_o Cd} \, ,$$

and their ratio is as follows:

$$\frac{E_y(d)}{H_y(d)} = \frac{{}_\parallel R_\perp \; {}_\parallel\bar{R}_\parallel \left(1 + {}_\perp\bar{R}_\perp\right)}{\left(1 - {}_\perp\bar{R}_\perp \; {}_\perp R_\perp\right)\left(1 + {}_\parallel\bar{R}_\parallel\right)} \equiv \gamma(d) \, . \tag{28}$$

In equation (28) the reflection coefficients are evaluated at $z = d$.

3. WAVEGUIDE MODE THEORY – HORIZONTALLY INHOMOGENEOUS GUIDES

If significant variations occur (as they often do) along the direction of propagation in the earth-ionosphere waveguide, then homogeneous guide formulations require modification. For such cases, there are two techniques for mode summation that are commonly used at VLF. One employs a WKB approach; the other uses mode conversion.

In the WKB technique, an eigenmode is assumed to be uniquely and independently identifiable anywhere along a propagation path. Furthermore, each mode is assumed to depend only on the local characteristics of the waveguide and to propagate independently of the existence of any other modes. The WKB formulation generalizes the expression for the mode summation in equation (1) to the form

$$E_v(d) = \frac{K(P,f)}{\sqrt{\sin d/a}} \ \Sigma_n \left(\Lambda_n^T \ \Lambda_n^R\right)^{1/2} \ G_n^T G_n^R \ e^{-ik_o \int_o^d S_n(x)dx} \ . \tag{29}$$

Note that the only differences arise from (1) the use of the geometric mean of the excitation factors at transmitter $\left(\Lambda_n^T\right)$ and from receiver $\left(\Lambda_n^R\right)$ locations and (2) the integration of the complex propagation factor along the path. The WKB formulation was apparently introduced simply as an "obvious extension" of the homogeneous mode sum by Wait (1964), who further noted that the form is similar to what would be expected if a WKB formulation were applied, although no actual derivation was presented. The use of the name WKB mode sum has persisted.

In the mode conversion method, eigenmodes are assumed to be uniquely and independently identified only in a local sense. Even though the eigen characteristics of a mode depend only on local characteristics of the waveguide, the propagation of each mode depends not only on the existence of other modes but also on the past and future propagation history of itself and the other modes.

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The approach that uses mode conversion does not lend itself conveniently to consideration of transition fading on very long VLF propagation paths. The WKB method, although easily applied, is unable to handle the rapid change in guide characteristics that occurs at the terminator (Pappert and Snyder, 1972). A difficulty encountered when using mode conversion arises from the necessity of knowing the future propagation history of each mode. Making appropriate assumptions to relieve this requirement can lead to some simplification, but the resulting formulation is still not convenient for very long paths. A further difficulty is that conventional mode conversion techniques require a knowledge of all the vector components of the electromagnetic field throughout all space.

Numerous methods have been presented for circumventing the latter problem. Wait (1968a, b), for example, formulated mode conversion by using isotropic impedance boundary conditions, with the result that fields were required only for the region between the upper (ionospheric) and the lower (ground) impedance boundaries. Such boundary conditions are certainly not appropriate for the nighttime ionosphere and thus for consideration of transitions from night to day. Smith (1974) extended Wait's (1968a, b) results to include boundary anisotropy by using impedance matrix boundary conditions. His analysis however, was for a planar, empty waveguide and was directed toward considering ground effects.

Galejs (1971) presented a modified mode conversion formulation for an abrupt change between daytime and nighttime anisotropic ionospheres. Conventional mode conversion theory usually represents the EM field as an expansion in terms of a suitably chosen complete set of orthogonal functions. The Galejs formulation employs an incomplete set of nonorthogonal functions. The formulation now to be employed is similar to that of Galejs (1971).

OPERATOR EQUATIONS

The operator equations presented in this section are based on Maxwell's equations in two (rectangular) dimensions and are patterned after the formulations of Pappert and Smith (1972). Furthermore, material media are assumed to be characterized by constitutive parameters (permittivity and permeability) that can be inhomogeneous in one dimension. This is quite compatible with the usual procedure for handling the earth-ionosphere waveguide by describing the

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electromagnetic properties of the ionosphere with the aid of a vertically inhomogeneous dielectric tensor.

We start with equation (5), use the substitutions in equation (6), and eliminate the E_x and H_x components. This leads to the differential operator equation

$$\underset{\sim}{L} \underset{\sim}{e} = -\frac{1}{ik_o} \frac{\partial \underset{\sim}{e}}{\partial x},$$

(30)

where $\underset{\sim}{e}$ is a four-element column vector:

$$\underset{\sim}{e} = \begin{pmatrix} E_y \\ E_z \\ H_y \\ H_z \end{pmatrix},$$

(31)

and $\underset{\sim}{L}$ is a 4 x 4 matrix differential operator. Equation (30) is cast as an eigenvalue equation by using the functional form $\exp\left[-ik_o S_n x\right]$, so that $\underset{\sim}{L} \underset{\sim}{e}_n = S_n \underset{\sim}{e}_n$. These $\underset{\sim}{e}_n$ form the set of expansion functions used in conventional mode conversion theory.

To find an operator adjoint to $\underset{\sim}{L}$, a procedure discussed by Pappert and Smith (1972) is used. First, let the inner product be defined:

$$\langle \underset{\sim}{a}, \underset{\sim}{b} \rangle = \int_{-\infty}^{\infty} \underset{\sim}{\tilde{a}}{}^* \cdot \underset{\sim}{b} \, dz,$$

(32)

where the tilde over $\underset{\sim}{a}$ denotes transpose and the asterisk denotes conjugation. Next, the operator adjoint to $\underset{\sim}{L}$ is defined by the relationship

$$\langle \underset{\sim}{L}{}^+ \underset{\sim}{w}, \underset{\sim}{e} \rangle \equiv \langle \underset{\sim}{w}, \underset{\sim}{L} \underset{\sim}{e} \rangle,$$

(33)

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which is just a prescription for integration by parts to find the form of L^+ as well as the necessary boundary conditions on the adjoint functions ($\underset{\sim}{w}$) in order that the integrated contributions vanish. Pappert and Smith (1972) showed that the adjoint functions can be obtained from direct functions by considering a second waveguide which is related to the original one in a simple way. In the original guide, the z-axis is taken as vertical and propagation is assumed in the xz-plane. The x-direction-cosine of the geomagnetic field is ℓ in the original guide. The adjoint guide is obtained by replacing ℓ with $-\ell$. By examining symmetry properties of the elements of the ionospheric permittivity tensor, Pappert and Smith (1972) showed that the adjoint eigenfunctions are given in terms of eigenfunctions of the adjoint waveguide according to the relationship

$$
\underset{\sim}{w}{}^*_n = \begin{pmatrix} w^*_{1n} \\ w^*_{2n} \\ w^*_{3n} \\ w^*_{4n} \end{pmatrix} = \begin{pmatrix} H_{zn}(-\ell) \\ -H_{yn}(-\ell) \\ -E_{zn}(-\ell) \\ E_{yn}(-\ell) \end{pmatrix} . \tag{34}
$$

Further, from the definition of adjoint given by equation (33) the usual biorthogonality relations hold. Thus

$$
\begin{aligned}
\langle L^+ \underset{\sim}{w}_n, \underset{\sim}{e}_m \rangle &= \lambda^*_n \langle \underset{\sim}{w}_n, \underset{\sim}{e}_m \rangle \\
&= \langle \underset{\sim}{w}_n, L \underset{\sim}{e}_m \rangle \\
&= S_m \langle \underset{\sim}{w}_n, \underset{\sim}{e}_m \rangle ,
\end{aligned}
$$

which shows both that $\langle \underset{\sim}{w}_n, \underset{\sim}{e}_m \rangle = 0$ and that $\lambda_n = S^*_n$ for nondegenerate eigenfunctions.

MODE CONVERSION

Fundamental to the mode conversion formulation employed here is the notion that any variation in the VLF propagation conditions along a path can

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be modeled as a series of discrete changes between homogeneous sections along the path. With this in mind, it is seen that the fundamental analytical requirement is the determination of mode conversion coefficients due to a single abrupt change in waveguide characteristics.

Following classical procedures, consider a mode of unit amplitude incident on such a discontinuity in a waveguide. This mode is described in terms of its height-gain function, $\vec{\underset{\sim}{e}}_j^{\ I}(z)$, where the superscript I denotes the incident region, the subscript j denotes the jth mode, the overarrow indicates a forward-propagating mode, and the undertilde indicates a vector quantity. Note that the terms height-gain function and eigenfunction are used here interchangeably even though more traditional usage limits the term height-gain to individual components of the vector eigenfunction. The discontinuity in the guide generates both reflected, or backward-propagating, modes within the incident region (I) and transmitted, or forward-propagating, modes within the transmission region (II). Backward-propagating modes might also exist in region II due to another discontinuity; these will therefore be included. The continuity requirements lead to the equation

$$\vec{\underset{\sim}{e}}_j^{\ I}(z) + \sum_m r_{mj} \overleftarrow{\underset{\sim}{e}}_m^{\ I}(z) = \sum_n T_{nj} \vec{\underset{\sim}{e}}_n^{\ II}(z) + \sum_l R_{lj} \overleftarrow{\underset{\sim}{e}}_l^{\ II}(z), \qquad (35)$$

which is independent of the position of the discontinuity between regions I and II. In equation (35), r_{mj} is a coefficient describing the conversion of the incident mode (j) into a backward-traveling mode (m) in region I $\left[\overleftarrow{\underset{\sim}{e}}_m^{\ I}(z) \right]$, and T_{nj} is a coefficient describing conversion of the incident mode (j) into the forward traveling mode (n) of region II $\left[\vec{\underset{\sim}{e}}_n^{\ II}(z) \right]$. The R_{lj} are the coefficients appropriate to possible backward-traveling modes in region II $\left[\overleftarrow{\underset{\sim}{e}}_l^{\ II}(z) \right]$.

Recall that a major problem with implementation of mode conversion concepts arises from the need to know the future propagation history. In terms of equation (35), this concept is represented by the coefficients R_{lj}. By making the assumption that appreciable reflection does not occur, so that there can be no backward-traveling modes, the R_{lj} may be set equal to zero. The r_{mj} are then also all zero, and equation (35) reduces to

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$$\vec{e}_j^I(z) = \sum_n T_{nj} \vec{e}_n^{II}(z). \qquad (36)$$

The assumption of negligible reflection is not unique to this study. It is commonly made and has been demonstrated to be adequate in numerous cases (Wait, 1968a, b; Galejs, 1971; Pappert and Snyder, 1972; Smith, 1974). In equation (36), there are assumed to be N modes in region II, so that there are N conversion coefficients.

Conventional mode conversion techniques require multiplication of equation (36) by the transpose conjugate of the adjoint *function for* the mth forward-traveling mode in region II, followed by integration over all space. Doing so, and recalling the biorthogonality relationships, we obtain the well-known result

$$\int_{-\infty}^{\infty} \tilde{w}_m^* \cdot \vec{e}_j^I \, dz = T_{mj} \int_{-\infty}^{\infty} \tilde{w}_m^* \cdot \vec{e}_m^{II} \, dz,$$

more conveniently written as

$$I_{m,j}^{II,I} = T_{mj} \, I_{m,m}^{II,II} \qquad (m = 1,N). \qquad (37)$$

Solving for the conversion coefficient yields the apparently simple result

$$T_{mj} = I_{m,j}^{II,I} / I_{m,m}^{II,II}. \qquad (38)$$

The simplicity of equation (37) depends on knowing the height-gain functions throughout all space. Such a requirement is the main limitation to the application of mode conversion techniques to very long VLF paths. Relaxing the integration limits to some finite value prevents use of the biorthogonality relationships and yields, instead of equation (37), the result

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$$I_{m,j}^{II,I} = \sum_n T_{nj} \, I_{m,n}^{II,II} \quad (m = 1,N), \tag{39}$$

where there are N equations and N unknowns, so that the system can be solved for the T_{nj}. In equation (39),

$$I_{m,j}^{II,I} = \int_\infty^h \tilde{w}_m^{*II} \cdot \underset{\sim}{e}_j^I \, dz$$

$$I_{m,n}^{II,II} = \int_\infty^h \tilde{w}_m^{*II} \cdot \underset{\sim}{e}_n^{II} \, dz, \tag{40}$$

where the overarrow notation is dropped because only forward-propagating modes are considered.

For application to the VLF earth-ionosphere problem, the above procedure may be justified as follows. First, the various eigenvalues and eigenfunctions are determined by using the complete geometry $(-\infty < z < +\infty)$. Second, the eigenfunctions can be expected to decay rapidly within the earth and within the ionosphere. Below the ionosphere but within the waveguide, the vector eigenfunctions degenerate into the isotropic TM and TE components, with a prescribed coupling between the two for a given mode. By taking the upper limit on the integral to be within the guide and below the ionosphere, it is possible to write the various integrals, $I_{n,k}$, in terms of known functions—namely the modififed Hankel functions, as defined by the Computation Laboratory Staff at Cambridge, Massachusetts (1945). The procedure used here is actually expansion in terms of "nonorthogonal" functions and is certainly better than assuming orthogonality and "throwing away" the contributions to the integrals from the ionosphere.

The jth vector eigenfunction for either region is defined to be [from equation (31)]

$$\underset{\sim}{e}_j = \begin{pmatrix} e_{yi} \\ e_{zj} \\ h_{yj} \\ h_{zj} \end{pmatrix}. \tag{41}$$

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Because the upper limit on the integrals in equation (40) is to be taken below the ionosphere, equation (41) can be written as

$$\underset{\sim}{e}_j = \begin{pmatrix} e_{yj} \\ -S_j h_{yj} \\ h_{yj} \\ S_j e_{yj} \end{pmatrix}.$$

The adjoint vector eigenfunction is given by equation (34) as

$$\underset{\sim}{w}_m = \begin{pmatrix} h^*_{zm}(-\ell) \\ -h^*_{ym}(-\ell) \\ -e^*_{zm}(-\ell) \\ e^*_{ym}(-\ell) \end{pmatrix} = \begin{pmatrix} S^*_m e^*_{ym}(-\ell) \\ -h^*_{ym}(-\ell) \\ S^*_m h^*_{ym}(-\ell) \\ e^*_{ym}(-\ell) \end{pmatrix},$$

where the notation indicates that elements of the adjoint eigenfunction are obtained by reversing the x-axis direction-cosine of the earth's biasing magnetic field. The first integral in equation (40) thus becomes

$$I^{II,I}_{m,j} = \left(S^{II}_m + S^I_j \right) \int_o^h \left[e^{II}_{ym}(-\ell)\, e^I_{yj} + h^{II}_{ym}(-\ell)\, h^I_{yj} \right] dz. \tag{42}$$

The normalization of the eigenfunctions is such that $h_{ym}(-\ell) = h_{ym}$. However, from equation (28)

$$\begin{aligned} \frac{e_y(-\ell)}{e_y} &= \frac{\gamma(d,-\ell)}{\gamma(d)} \\ &= \frac{{}_IR_\perp(-\ell)\left(1 - {}_I\bar{R}_\perp \,{}_IR_\perp\right)}{{}_IR_\perp\left(1 - {}_I\bar{R}_\perp \,{}_IR_\perp(-\ell)\right)}. \end{aligned} \tag{43}$$

Because $_\perp R_\perp(-\ell) = {}_\perp R_\perp$ and $_\parallel R_\perp(-\ell) = {}_\perp R_\parallel$ (Budden, 1961b), equation (43) becomes

$$\frac{e_y(-\ell)}{e_y} = \frac{_\perp R_\parallel}{_\parallel R_\perp} \ .$$

Denoting this ratio as A_m, equation (42) becomes

$$I_{m,j}^{II,I} = \left(S_m^{II} + S_j^I\right) \int_o^h \left[A_m\, e_{ym}^{II}\, e_{yj}^I + h_{ym}^{II}\, h_{yj}^I\right]\, dz. \tag{44}$$

The integrals in equation (44) are now easily evaluated by using the conventional "multiply and subtract" method. In this method, we multiply the differential equation for the height-gain of mode m by the height-gain of mode n and the differential equation for the height-gain of mode n by the height-gain of mode m and subtract. The differential forms are obtained from equation (9a, b) and will be presented shortly [see equation (50)]. Performing the multiplication and subtraction for h_y components leads to

$$h_{ym}\frac{d^2 h_{yn}}{dz^2} - h_{yn}\frac{d^2 h_{ym}}{dz^2} - k_o^2\left(C_m^2 - C_n^2\right) h_{ym} h_{yn} = 0,$$

or

$$\frac{d}{dz}\left(h_{ym}\frac{dh_{yn}}{dz} - h_{yn}\frac{dh_{ym}}{dz}\right) = k_o^2\left(C_m^2 - C_n^2\right) h_{ym} h_{yn}.$$

The second integral in equation (44) thus becomes

$$\int_o^h h_{ym}^{II} h_{yj}^I \, dz = \frac{h_{ym}^{II} \dfrac{dh_{yj}^I}{dz} - h_{yj}^I \dfrac{dh_{ym}^{II}}{dz} \bigg|_o^h}{k_o^2 (c_m^{II} - c_j^I)(c_m^{II} + c_j^I)}$$

$$= \frac{-i \left[h_{ym}^{II} e_{xj}^I - h_{yj}^I e_{xm}^{II} \right]_o^h}{k_o (c_m^{II} - c_j^I)(c_m^{II} + c_j^I)} , \qquad (45)$$

while the first integral in equation (44) is

$$\int_o^h e_{ym}^{II} e_{yj}^I \, dz = \frac{i \left[e_{ym}^{II} h_{xj}^I - e_{yj}^I h_{xm}^{II} \right]_o^h}{k_o (c_m^{II} - c_j^I)(c_m^{II} + c_j^I)} . \qquad (46)$$

In equations (45) and (46), it is assumed that $c_m^{II} \neq c_j^I$ for all m and j. Evaluation of the second integral in equation (40) proceeds in the same manner. The only difference is the fact that all field components in this case pertain to region II of the waveguide. In this case however, the situation $m = j$ results in division by zero in equations (45) and (46), and special treatment is required. For the case $m = j$ in region II, the use of L'Hospital's rule yields

$$\int_o^h h_{yj}^2 \, dz = \frac{1}{\alpha} \left[(c_j^2 + \alpha z) h_{yj}^2 - e_{xj}^2 \right]_o^h \qquad (47)$$

$$\int_o^h e_{yj}^2 \, dz = \frac{1}{\alpha} \left[(c_j^2 + \alpha z) e_{yj}^2 - h_{xj}^2 \right]_o^h , \qquad (48)$$

where the superscript "II" is omitted.

The contribution to the integral in equation (40) due to the fields in the ground is obtained as follows. The fields vary as $\exp(+ik_o \tau z)$ in the ground, which gives

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$$\int_{-\infty}^{o} h_{yj}^{I} h_{ym}^{II} dz = - \frac{i}{k_o \left(\tau_j^I + \tau_m^{II} \right)}$$

$$\int_{-\infty}^{o} e_{yj}^{I} e_{ym}^{II} dz = \frac{i \, \gamma_j^I \, \gamma_m^{II}}{k_o \left(\tau_j^I + \tau_m^{II} \right)} . \tag{49}$$

The individual height-gains are obtained in the following way. Consider the transformed planar guide already discussed under Coordinate Transformation. In order to be compatible with the assumptions discussed under Mode Equation and under Mode Excitation Factors and Polarization Coupling, the waveguide region is assumed to be filled with the "equivalent" permeability or dielectric medium as described in under Coordinate Transformation. The fields within the "filled" region satisfy equations (9a, b) while the fields within the earth have simple exponential forms.

Since all altitudes of interest are small compared to the radius of the earth (a), the exponential term in equation (9) is expanded to first order in z/a. A variation in the x-direction of the form $e^{-ik_o Sx}$ is assumed. Note that the variable S will be identified later as $\sin(\theta)$, where the angle θ is referenced to the vertical rather than to the horizontal as in previous sections. With the above assumptions, equation (9b) for TM waves becomes

$$H_y'' + k_o^2 \left(c^2 + \alpha z \right) H_y = 0, \tag{50}$$

where $c^2 = 1 - s^2$, $\alpha = 2/a$, and the double prime denotes second differentiation with respect to z.

With the change of variable

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$$t = \left[\frac{k_o}{\alpha} \right]^{2/3} \left(c^2 + \alpha z \right), \tag{51}$$

the equation becomes

$$H_y'' + t H_y = 0, \tag{52}$$

where now the double prime denotes differentiation with respect to t. Equation (52) is Stokes' equation. Thus, H_y is linearly expressible in terms of solutions to Stokes' equation. In this study, we shall use the modified Hankel functions of order one-third (h_1, h_2), as defined by the Computation Laboratory Staff at Cambridge, Massachusetts (1945). Using a similar procedure for TE polarization, we shall arrive at similar results except that the transformed planar guide will contain a medium with refractive index given by $e^{z/a}$.

The waves in the two regions are given by the following relationships:

Region I:
z > d

$$H_y = a_1 h_1(t) + a_2 h_2(t)$$

$$E_x = i \left[\frac{\alpha}{k_o} \right]^{1/3} \left[a_1 h_1'(t) + a_2 h_2'(t) \right]$$

Region II:
z > 0

$$H_y = e^{i k_o \tau z}$$

$$E_x = - \frac{\tau}{N^2} e^{i k_o \tau z},$$

where h_1 and h_2 are modified Hankel functions of order one-third, N is the refractive index of the ground, and $\tau = \left(N^2 - S^2\right)^{1/2}$, taken with negative imaginary part. Continuity requirements between the regions leads to the results

$$H_y = f_\parallel(z) \ ,$$

where

$$f_\parallel(z) = H_2\left(t_o\right) h_1\left(t_z\right) - H_1\left(t_o\right) h_2\left(t_z\right)$$

$$H_j\left(t_z\right) = h_j{}'\left(t_z\right) - i \left[\frac{k_o}{\alpha}\right]^{1/3} \frac{\tau}{N^2} h_j\left(t_z\right)$$

$$t_z = \left[\frac{k_o}{\alpha}\right]^{2/3} \left(c^2 + \alpha z\right) \tag{53}$$

Following a similar procedure, results for the TE polarization are given:

$$E_y = f_\perp(z),$$

where

$$f_\perp(z) = F_2\left(t_o\right) h_1\left(t_z\right) - F_1\left(t_o\right) h_2\left(t_z\right)$$

$$F_j\left(t_z\right) = h_j'\left(t_z\right) - i \left[\frac{k_o}{\alpha}\right]^{1/3} \tau h_j\left(t_z\right). \tag{54}$$

Normalizing the H_y component to 1 at $z = 0$ and using equation (28) for the polarization coupling, we arrive at the final forms for the height gains:

$$h_y(z) = \frac{f_\parallel(z)}{f_\parallel(0)} \tag{55a}$$

$$e_y(z) = \gamma(d) \frac{f_\parallel(d)}{f_\parallel(0)} \frac{f_\perp(z)}{f_\perp(d)} \tag{55b}$$

$$e_x(z) = i \left(\frac{\alpha}{k_o}\right)^{1/3} \frac{1}{f_\parallel(0)} \frac{df_\parallel(z)}{dt} \tag{55c}$$

$$h_x(z) = -i \left(\frac{\alpha}{k_o}\right)^{1/3} \frac{\rho(d) f_\parallel(d)}{f_\parallel(0) f_\perp(d)} \frac{df_\perp(z)}{dt} \tag{55d}$$

$$e_z(z) = -S h_y(z) \tag{55e}$$

$$h_z(z) = S\gamma(d) h_y(z). \tag{55f}$$

Once the conversion coefficients are obtained, it is necessary to sum over all the incident modes. This requires evaluation of the amplitudes of the incident modes. This can be done by starting at the transmitter, where the modal excitation factors give the mode amplitudes, and by assuming that the regions between the discontinuities are homogeneous.

Notationally this can be written as

$$\underset{\sim}{A}^I = \underset{\sim}{P}^I \underset{\sim}{A}^I, \tag{56}$$

where the equation

$$\underset{\sim}{A}^I = \begin{matrix} A_1^I \\ \vdots \\ A_j^I \end{matrix}$$

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represents the amplitudes of the incident modes $\left(A_j^I\right)$ at the end of the region I and the equation

$$a^I = \begin{array}{c} a_1^I \\ \cdot \\ \cdot \\ a_j^I \end{array}$$

represents the amplitudes of the modes at the start of region I. The factor p^I is a diagonal matrix with the nth diagonal element given as follows:

$$P_n = e^{-ik_o S_n^I (\Delta\rho)^I}. \tag{57}$$

Note that P_n represents the propagation factor appropriate to a homogeneous section of length $(\Delta\rho)^I$, assumed to be the length of region I. Note that the amplitudes of the modes in region II just after conversion can be conveniently written as

$$a^{II} = T A^I = T P^I a^I, \tag{58}$$

where T is a rectangular matrix of the conversion coefficients from region I into region II.

A mode conversion formulation has been presented by Pappert and Shockey (1974), but important differences exist between their treatment and that employed here. Differences occur both in the numerical procedures employed and in the definition of the conversion coefficients. In the present formulation a conversion coefficient is computed at each slab interface independently of any previous conditions. In the Pappert formulation, the conversion coefficients are defined and computed in such a way that the effect of the propa-

gation factor [equation (57)] is included. Both definitions of conversion coefficients are reasonable, although their numerical values can be quite different. For example, first consider an isolated terminator specified as an abrupt transition. The two formulations will give the same results for the conversion coefficients. Next suppose that the terminator has a substantial extent. The conversion coefficients then differ for each slab interface after the first because the Pappert formulation includes the accumulated effects of propagation through the transition region, whereas the current formulation does not. In any case, the resulting mode sums computed by means of either formulation will be sensibly the same, with differences primarily due to computer numerics.

The Pappert definition of conversion coefficients is useful for studying a horizontal inhomogeneity that exists over a small distance, such as the dawn-dusk terminator. A similar conversion coefficient can be derived from the mode constants and the mode conversion coefficients presented in this study. What is required is the complex amplitude of a mode at the end of such a confined inhomogeneity resulting from a single mode of unit amplitude at the start of the inhomogeneity.

Denote by region 1 the region prior to the start of the *inhomogeneity*. The complex amplitudes of the modes in the first region of inhomogeneity (called region 2) due to a mode of unit amplitude and number k in region 1 are given by the conversion coefficients $T_{m,k}^{2,1}$ [see equation (44)]. The complex amplitudes of the modes at the end of region 2 are $A_{m,k}^{2}$, where

$$A_{m,k}^{2} = P_{m}^{2} \ T_{m,k}^{2,1}$$

and P_{m}^{2} is the propagation factor for mode m in region 2 [see equation (57)]. The complex amplitude of mode n at the start of the next region (region 3) due to a mode of unit amplitude in region 2 is $T_{n,m}^{3,2}$, so that the actual complex amplitude of the mode n in region 3 is given by the product of $T_{n,m}^{3,2}$ and . Summation over all the modes of region 2 gives the total complex

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amplitude of mode n. Thus

$$a_{n,k}^3 = \sum_m A_{m,k}^2 \; T_{n,m}^{3,2} \; P_m^2 \; T_{m,k}^{2,1}. \qquad\qquad (59)$$

This result can be generalized to any inhomogeneity along the path. We obtain

$$a_{m,k}^p = \sum_n T_{m,n}^{p,p-1} \; P_n^{p-1} \; a_{n,k}^{p-1}. \qquad p > 3 \qquad (60)$$

The formulations presented in this section have been implemented in a computer program written in FORTRAN. The development of the numerical cal- culations required for the reflection coefficients and waveguide modes has occurred over many years. These efforts have been supported by the Defense Nuclear Agency, NAVELEX PME 117, the Defense Communications Agency, and the US Coast Guard.

4. FASTMC

FASTMC is a computer program which employs the approximate mode conversion model described in section 3. The program is written in Univac's version of ASCII FORTRAN. It accepts card image input and generates plots. Additional outputs available are printouts of the amplitude and phase and binary output of the complex field. The source is an arbitrarily oriented dipole at any altitude within the waveguide. The vertical (e_z) and horizontal broadside (e_y) fields at any altitude can be obtained.

Input to the program is controlled by a set of control cards which define the type of data being read and allow standard defaults to apply. These control cards are described below. All eighty columns of these cards are read and printed. However, only the first four columns are examined by the program. Thus, the input card and the printout can contain additional comment. For example, "NAMELIST DATA FOR ELEVATED TRANSMITTER" can be used in place of the minimum requirement of "NAME". The control cards are described below, and a sample set is shown in figure 1.

NAME   Signals that NAMELIST data follow. In this program the NAMELIST name is DATUM. The NAMELIST variables are described below.

ICOMP   = Index of received electric field component
                  = 1 gives the vertical field (Z) - DEFAULT
                  = 2 gives the horizontal transverse field (Y)

TALT   = Transmitter altitude in km - DEFAULT = 0.0

RALT   = Receiver altitude in km - DEFAULT = 0.0

INCL   = Inclination of transmitting antenna with respect to the vertical in degrees - DEFAULT = 0.0.

THETA   = Orientation of transmitting antenna's projection in the horizontal plane with respect to the direction of propagation in degrees - DEFAULT = 0.0.

```
NAME
 &DATUM
 NPRINT=3,
 DMAX=3., SIZEX1=6.,
 &END
DATA
FASTMC TEST, D=0
R  0.0    F 31.1350 A 208.234 C    4.388 M 5.600D-05 S 4.640D 00 E 81.0
1 89.78599 -6.258922-8.01755519D-06-3.14370439D-06-1.75676358D-12-6.23796911D-13
2 89.78599 -6.258922-3.66885085D-09-1.80621118D-09 9.48673970D-01-1.86114244D-01
1 88.98413 -1.682941 4.55299003D-03-1.98787729D-02-2.71479704D-12 2.65511284D-12
2 88.98413 -1.682941-2.44034593D-07-1.55170614D-07 9.59499762D-01-1.69936355D-01
1 83.31784 -0.608891 1.25067533D-03-2.33661345D-02-3.81565960D-12 2.18502245D-12
2 83.31784 -0.608891-2.60604588D-07-1.90988579D-07 9.71503132D-01-1.46315500D-01
1 78.98910 -0.665431 9.73001634D-04-2.02983105D-02-1.55572876D-11-3.42740333D-12
2 78.98910 -0.665431-3.22053335D-07-4.71566890D-07 9.90195350D-01-1.04247840D-01
1 74.97312 -0.764111 1.23487260D-04-1.96782005D-02-1.08614545D-11-2.84683435D-11
2 74.97312 -0.764111-1.19189454D-07-7.55284703D-07 1.01070238D 00-3.73626434D-02

R  1.000 F 31.1350 A 208.234 C    4.388 M 5.600D-05 S 1.000D-05 E   5.0
1 89.68035 -5.566901-1.24182979D-05 1.27807499D-04 7.36555429D-09 2.50792168D-08
2 89.68035 -5.566901 4.14210291D-09-1.85664539D-06 9.50734751D-01-1.82150703D-01
1 86.05721 -1.194791 4.48635991D-04 1.68249745D-03 7.85507546D-08 2.61802701D-07
2 86.05721 -1.194791-6.50431216D-06-2.14721196D-05 9.61684773D-01-1.60169797D-01
1 80.93273 -1.269851 2.84383129D-03 5.42606425D-03 2.44905544D-07 5.71788780D-07
2 80.93273 -1.269851-2.27162120D-05-5.70098045D-05 9.74648413D-01-1.23476674D-01
1 76.61480 -1.636121 1.08225111D-02 1.10991352D-02 1.93580552D-08 8.07364329D-07
2 76.61480 -1.636121-4.01520636D-05-1.05085694D-04 9.87201164D-01-6.43089003D-02
1 72.32836 -2.025471 3.26814733D-02 6.00031025D-03-5.46015651D-07 7.30226214D-07
2 72.32836 -2.025471-6.72016623D-05-1.61624764D-04 9.87170083D-01 2.67031249D-02

R  1.500 F 31.1350 A 208.234 C    4.388 M 5.600D-05 S 4.640D 00 E 81.0
1 89.78599 -6.258922-8.01755519D-06-3.14370439D-06-1.75676358D-12-6.23796911D-13
2 89.78599 -6.258922-3.66885085D-09-1.80621118D-09 9.48673970D-01-1.86114244D-01
1 88.98413 -1.682941 4.55299003D-03-1.98787729D-02-2.71479704D-12 2.65511284D-12
2 88.98413 -1.682941-2.44034593D-07-1.55170614D-07 9.59499762D-01-1.69936355D-01
1 83.31784 -0.608891 1.25067533D-03-2.33661345D-02-3.81565960D-12 2.18502245D-12
2 83.31784 -0.608891-2.60604588D-07-1.90988579D-07 9.71503132D-01-1.46315500D-01
1 78.98910 -0.665431 9.73001634D-04-2.02983105D-02-1.55572876D-11-3.42740333D-12
2 78.98910 -0.665431-3.22053335D-07-4.71566890D-07 9.90195350D-01-1.04247840D-01
1 74.97312 -0.764111 1.23487260D-04-1.96782005D-02-1.08614545D-11-2.84683435D-11
2 74.97312 -0.764111-1.19189454D-07-7.55284703D-07 1.01070238D 00-3.73626434D-02

R 40.
NAME
 &DATUM NPRINT=1, TALT=6., RALT=6., INCL=90., THETA=90., &END
START
NAME
 &DATUM INCL=0., THETA=0., NPPLOT=1, NPRINT=0, ICOMP=2, &END
START
NAME
 &DATUM INCL=90., THETA=90., &END
START
```

Figure 1. Sample card deck for FASTMC.

TOPHT   = Height of the bottom of the ionosphere in km [integration limit h in equations (49) - (53)] - DEFAULT = 90.0

NPRINT   = Flag to control printout of mode conversion and mode sum results
= 0 gives no printout
= 1 gives printout of mode parameters and calculated fields vs. distance - DEFAULT
= 2 also prints the input mode constants cards and the mode conversion coefficients
= 3 also prints the integrals used in the calculation of the mode conversion coefficients.

NAPLOT   = Flag to control amplitude plots
= 0 gives no plot of amplitude
= 1 gives amplitude plots - DEFAULT

AMPMIN   = Minimum amplitude to be plotted in dB above one microvolt/meter - DEFAULT = 10.0.

AMPMAX   = Maximum amplitude to be plotted - DEFAULT = 70.0

AMPINC   = Tic mark interval (for plot border) - DEFAULT = 10.0

NPPLOT   = Flag to control plots of phase relative to the speed of light
= 0 gives no plot - DEFAULT
= 1 gives phase plots

PHSMIN   = Minimum phase excursion in degrees - DEFAULT = 360.0

PHSMAX   = Maximum phase excursion - DEFAULT - 360.0

PHSINC   = Tic mark interval (for plot border) - DEFAULT = 90.0

DMIN   = Minimum distance from the transmitter in Mm - DEFAULT =
                  0.0

DMAX   = Maximum distance - DEFAULT = 10.0

XINC   = Horizontal tic mark intervals (for plot border) - DEFAULT
                  = 1.0

SIZEX   = Length of horizontal axis in inches - DEFAULT = 10.0

SIZEY   = Length of vertical axis. Maximum value is 9.0 - DEFAULT
                  = 6.0

NRCURV   = Number of sets of data to be plotted in one graph.
                  Maximum value is 4 - DEFAULT = 1

NRPTS   = Number of data points to be calculated. Maximum value is
                  501 - DEFAULT = 501

TOTAPE   = Binary output flag
                = 0 gives no output - DEFAULT
                = 1 writes the complex mode sum and path parameters to
                  logical unit 2

TLONG,TLAT   = Transmitter coordinates in degrees west and north -
                  DEFAULT = 0.0, 0.0

RBEAR   = Path bearing in degrees clockwise from north - DEFAULT =
                  0.0

After reading the NAMELIST the program reads the next control card.

DATA   Signals that the propagation path data follow. The format
                of this data is that which is produced by MODESRCH (Morfitt
                and Shellman, 1976), WAVEGUID (Pappert et al, 1970) and the
                INTEGRATED PREDICTION PROGRAM (Ferguson, 1972). The first

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card contains the data set identification. This card is
used to supply a literal path description such as "HAWAII
TO WAKE". This is followed by sets of mode constants, one
for each path segment or slab. The first card for each
segment contains the starting distance (ρ), frequency (f),
magnetic azimuth, codip and intensity, ground conductivity
and permittivity (σ, ε), and the nominal height of the free
space portion of the waveguide (h). Of these parameters,
ρ, f, σ, and ε must appear. The magnetic parameters are
included for reference. The value of h may be included if
it is to vary among the segments. Otherwise it is a con-
stant value input via the NAMELIST. This card is followed
by the mode constants cards, one pair for each mode, con-
taining the following information:

$$
\begin{array}{ccccc}
1 & \theta & I & T_1 & T_2 \\
2 & \theta & I & T_3 & T_4
\end{array}
$$

where 1 and 2 are sequencing indices, θ is the complex
eigenangle at the ground, I is an index indicating the
functional form of the ratio e_y/h_y [equation (28)] and the
Ts are complex constants described below.

The list of modes for each segment is terminated by a blank
card. A maximum of 20 modes will be used although there is
no maximum number which the data deck may contain. The
list of segments is terminated by a card with $\rho = 40$.
Parameters for each segment are stored sequentially on
logical file 4. The number of path segments is limited by
the space allocated to this file. Each segment requires
1848 words of storage. After reading these data the program
generates the required calculations (as specified by the
NAMELIST input or defaults) then returns for another con-
trol card.

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BUMP   Signals that this card contains an overall plot iden-
        tification. It is used for additional information not
        contained in the data set identification cards. When
        several sets of data are plotted on one graph, it can be
        used to provide a connection between them.

START   Signals calculations using new NAMELIST parameter values
        for the current propagation path data.

A summary of how the control cards are used in FASTMC is shown in figure
2. A brief description of the subroutine functions is given in table 1, and
complete listings can be found in appendix A. Sample plotted outputs are
shown in figures 3 and 4. The Ts on the data cards are given by the following
relationships:

$$T_1 = \frac{\left(1 + {}_\parallel \bar{R}_\parallel\right)^2 \left(1 - {}_\perp R_\perp \, {}_\perp \bar{R}_\perp\right) s^{1/2}}{\frac{\partial F}{\partial \theta} \, {}_\parallel \bar{R}_\parallel \, f_\parallel{}^2(d)}$$

$$T_2 = \frac{\left(1 + {}_\perp \bar{R}_\perp\right)^2 \left(1 - {}_\parallel R_\parallel \, {}_\parallel \bar{R}_\parallel\right) s^{1/2}}{\frac{\partial F}{\partial \theta} \, {}_\perp \bar{R}_\perp \, f_\perp{}^2(d)}$$

$$T_3 = \frac{\left(1 + {}_\parallel \bar{R}_\parallel\right)\left(1 + {}_\perp \bar{R}_\perp \, {}_\parallel R_\parallel\right) s^{1/2}}{\frac{\partial F}{\partial \theta} \, f_\parallel(d) \, f_\perp(d)}$$

$$T_4 = \frac{{}_\perp R_\parallel}{{}_\parallel R_\perp} \, .$$

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(A)—— Read and print input card

    IF('NAME')————————————————————————

    IF('BUMP')————————————————

    IF('STAR')————————————   Read and print DATUM

    IF('DATA')————

    ERROR EXIT   (A)

    Read path identification   Read and store identification

—Read c, f, A, C, M, σ, ϵ, h

    IF($\rho = 40$)————————————————   (A)

—Read θ, I, T_1, T_2

    IF($\theta = 0$)————————————

Read θ, I, T_3, T_4

Calculate and store mode parameters

MCSTEP: Calculate and store conversion coefficients

MCFLDS: Calculate and store mode sum vs distance

MCPLTS: Print/plot mode sum

(A)
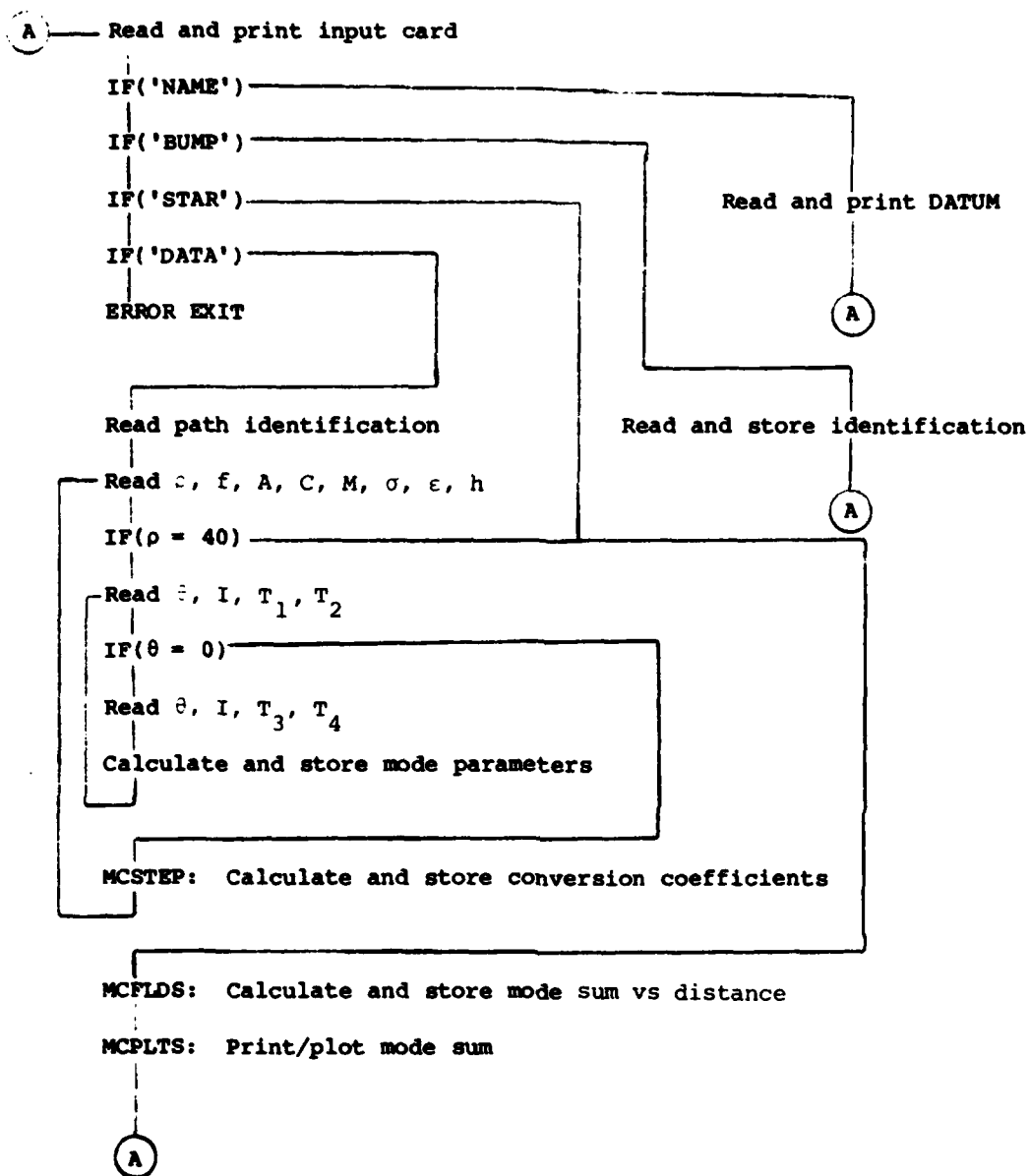
Figure 2. Illustration of FASTMC program execution sequence.

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MCSTEP   Calculates the integrals (50) and solves for the conversion
          coefficients (44) at each segment boundary. Calls MDHNKL
          and CLINEQ.

MCFLDS   Calculates the field strength from DMIN to DMAX. Calls
          HTGAIN.

MCPLTS   Prints and plots amplitude and phase. Calls BORDER, CURVE,
          and CALCOMP routines.

MDHNKL   Calculates modified Hankel functions of order one-third and
          their derivatives.

HTGAIN   Calculates height-gain functions (32, 33). Calls MDHNKL.

CLINEQ   Solves system of equations for complex coefficients.

BORDER   Draws border for plots. Labels minimum and maximum values
          of axes.

CURVE   Draws a curve of x vs y. Seven different line types are
          available. FASTMC uses only the first four types: solid,
          long, medium, and short dashes.

Table 1. FASTMC subroutines.

70.0

10.0
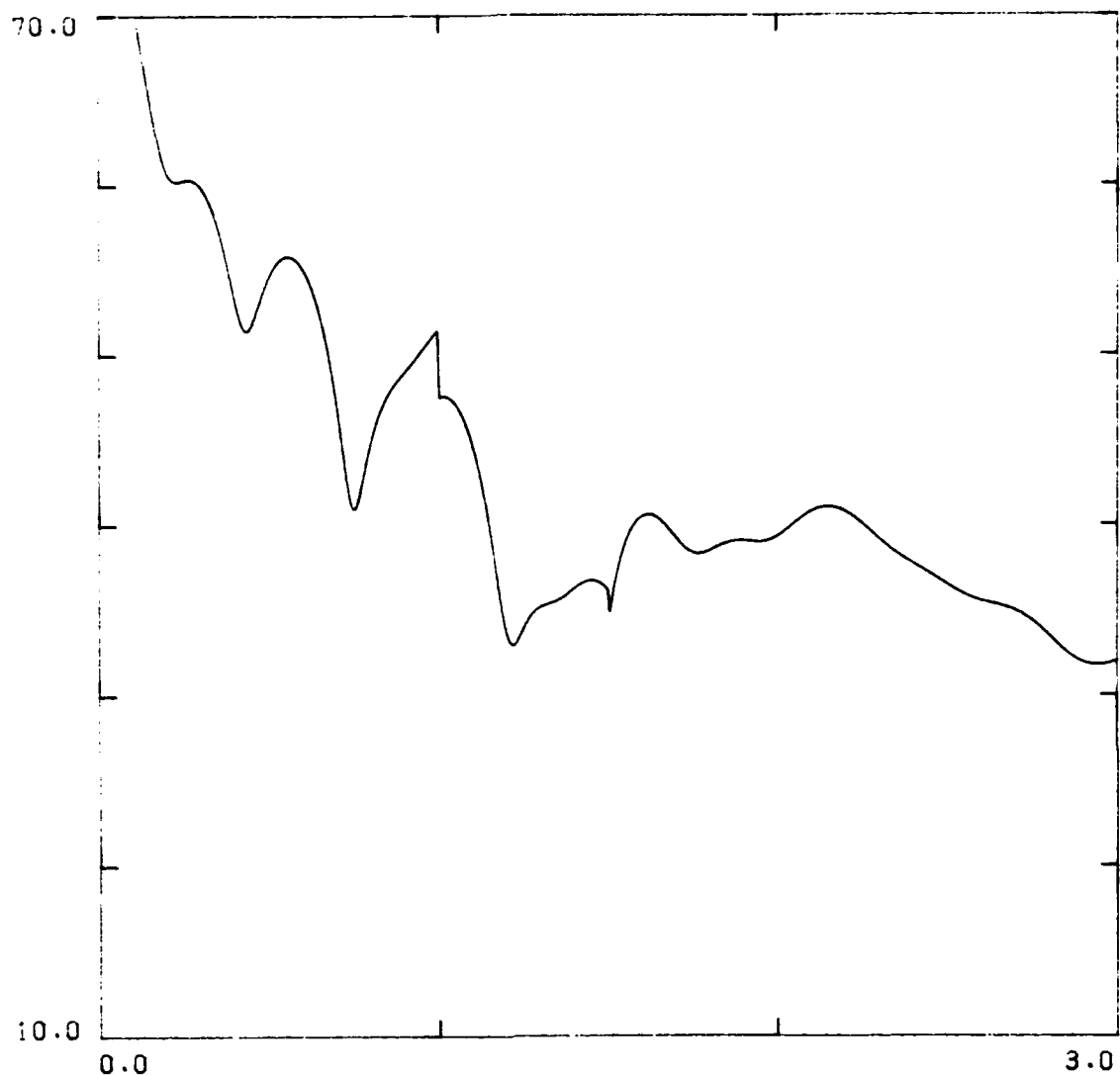
0.0   3.0

FMC 1
Z COMPONENT AMPLITUDE POWER = 1.   FREQ = 31.135
INCL = 0.   THETA = 0.   TALT = 0.0   RALT = 0.0

———————— FASTMC TEST, D=0

Figure 3.   Sample plot of amplitude output from FASTMC. This is
the first plot generated by the card deck of figure 1.

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Figure 4. Sample plot of relative phase output from FASTMC. This is the fourth plot generated by the card deck of figure 1.

From these relationships, equation (26) can be written as follows:

$$H_y = - \frac{k_o^{-5/2} e^{-i\pi/4} S}{2\epsilon_o \sqrt{2\pi} a \sin(\rho/a)} T_1 f_{\parallel}(z_T) f_{\parallel}(z_R) e^{-ik\rho S}, \quad (61)$$

and equation (28) can be evaluated at the ground, giving the relationship

$$\gamma = \frac{e_y}{h_y} = \frac{\left(1 + {_\perp}\overline{R}_\perp\right) {_\parallel}\overline{R}_\parallel {_\parallel}R_\perp}{\left(1 - {_\perp}\overline{R}_\perp {_\perp}R_\perp\right)\left(1 + {_\parallel}\overline{R}_\parallel\right)} \frac{f_{\parallel}(d)}{f_{\perp}(d)} = \frac{T_3}{T_1}, \quad (62)$$

where $f_{\parallel}$, $f_{\perp}$ are from equations (53) and (54).

The relationship in equation (62) is used if the value of I on the mode cards is 1. If the value of I is 2, the following relationship is used:

$$\gamma = \frac{\left(1 + {_\perp}\overline{R}_\perp\right)\left(1 - {_\parallel}\overline{R}_\parallel {_\parallel}R_\parallel\right)}{\left(1 + {_\parallel}\overline{R}_\parallel\right) {_\perp}\overline{R}_\perp {_\perp}R_\parallel} \frac{f_{\parallel}(d)}{f_{\perp}(d)} = \frac{T_2}{T_3 T_4}. \quad (63)$$

Equations (62) and (63) are identical if the value of the mode equation (12) is exactly zero. The value of I is determined from the mode polarization as calculated in the WAVEGUID, MODESRCH, or INTEGRATED PREDICTION PROGRAM. A mode which is principally TM will have $1 - {_\perp}R_\perp {_\perp}\overline{R}_\perp$ very small and the ratio γ will be very large, resulting in the use of equation (62). Conversely, a mode

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which is principally TE will have $1 - {}_\parallel R_\parallel \; {}_\parallel \overline{R}_\parallel$ very small, and the ratio γ will be very small, resulting in the use of equation (63).

The excitation factor for H_y due to a vertical radiator can be defined from equation (61) as $-ST_1$. Excitation factors due to horizontal end-fire and horizontal broadside radiators can be obtained from Pappert et al, 1970. In terms of the Ts, these are T_1 and $T_3 T_4$ for end-fire and broadside, respectively. The corresponding electric field components are obtained from equation (55).

5. BUMP

BUMP is a computer program which has been written to allow for a systematic variation of a path perturbation, with full allowance for variation of the mode parameters with conductivity and magnetic field. The most common perturbation is the day-night terminator. Previous studies of the effect of the terminator on propagation were limited to waveguides for which the magnetic field and ground conductivity were constant. BUMP allows for the study of the effect of the terminator along a path with mixed ground conductivities and/or significant magnetic field variations along the propagation path. The formulation of the program allows for other perturbations to be studied.

The design of the program was aimed at requiring the simplest possible data deck preparation. This was done in order to make it easy to modify propagation path data and to allow flexibility in the ionospheric models. The resulting input requires propagation path data for a series of horizontally homogeneous ionospheres. The data for each ionosphere are merged by the program to produce the required ionospheric perturbation. This program has been used to study models of the ionospheric variation from middle to high latitudes and to study trans-terminator propagation across the magnetic equator. The program to be described below is set up for a minimum path segment length of 5 km and a maximum path length of 40 Mm. This requires 801 random access records per ionosphere. Since a substantial portion of the cost of running the program is incurred in setting up the random access files required, users can achieve some cost savings by changing the 5 km and 40 Mm figures and modifying the random access files accordingly. A maximum of 30 modes can be stored for each path segment, requiring 1229 words of storage in each random access record.

Preparation of the input data for this program is illustrated for a bump in the ionosphere. The lowest height of ionosphere will be characterized by $h' = h_o$ and the highest height by $h' = h_6$. The variation of the ionosphere from lowest to highest is modeled by a linear variation of h' over six steps denoted by h_1, h_2, h_3, h_4, h_5. The length of each step is $\Delta \rho_1$ and the length of the step at h_6 is $\Delta \rho_2$. The bump in the propagation path is defined in figure 5.
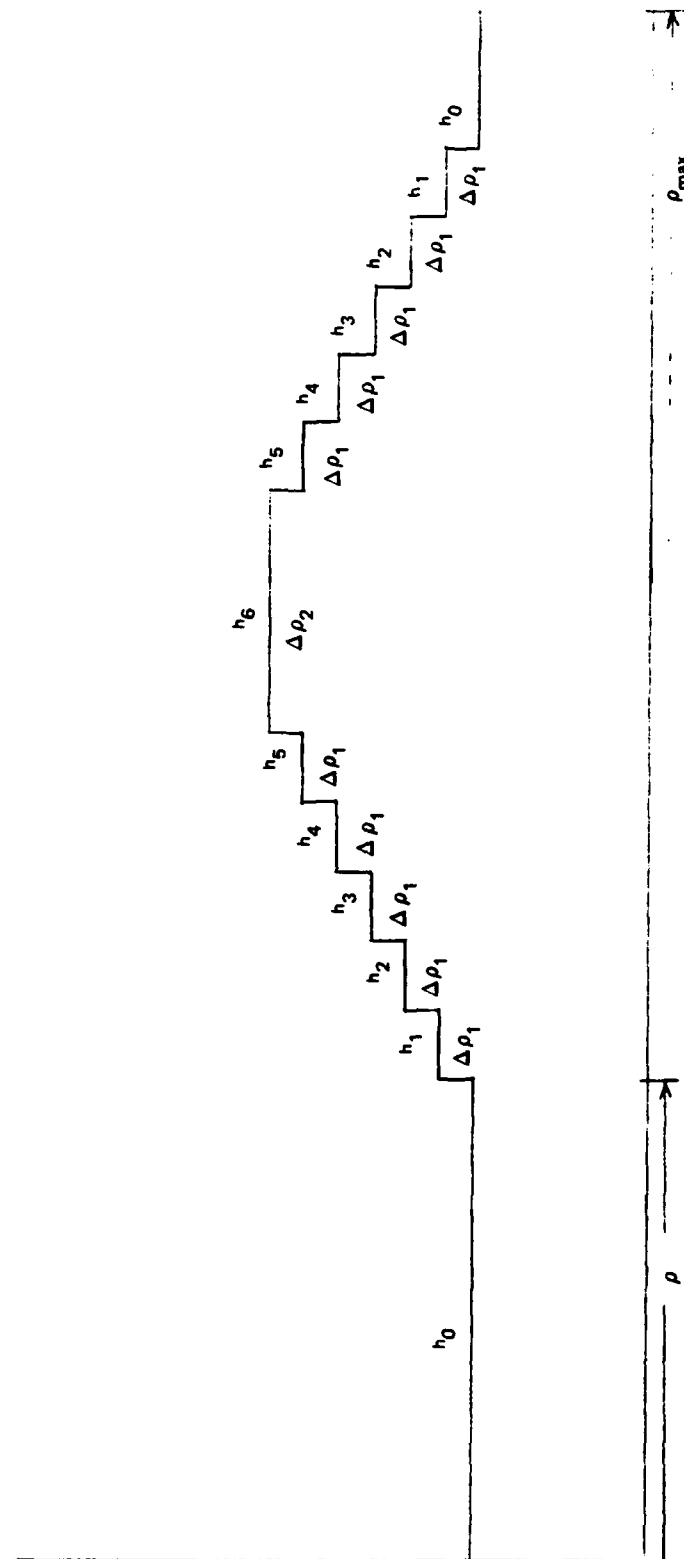
Figure 5. Illustration of BUMP model parameters.

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By making $\Delta\rho_2$ equal to 20 Mm, the bump becomes a ramp for modeling the day-night transition. If the order of input of the data sets corresponding to each h' were reversed, the ramp would model the night-day transition. Movement of the bump is accomplished by varying ρ from ρ_0 to ρ_L in increments of $\Delta\rho$. If ρ_0 is greater in value than ρ_L, the sign of $\Delta\rho$ is adjusted automatically. The total path length is ρ_{max}.

If the data set for h_n does not have data corresponding to $\rho + (n - 1)$ $\Delta\rho_1$, the data from the next smaller distance is used. For example, suppose h_n has data at 1.0 Mm and 1.2 Mm and $\rho + (n - 1)\Delta\rho_1$ is greater than 1.0 Mm and less than 1.2 Mm. The program will select the data for 1.0 Mm to be used at $\rho + (n - 1)\Delta\rho_1$. If the value of ρ becomes less than zero, the necessary steps in h_n are removed. For example, if $\rho = -1.5\Delta\rho_1$, the first step on the propagation path will be h_2. A similar procedure is followed if $\rho + (n - 1)\Delta\rho_1$ becomes greater than ρ_{max}.

The input to BUMP is a series of FASTMC compatible data sets, one for each ionospheric profile. Each data set must contain data for the required propagation path. The output from BUMP is a series of FASTMC data sets, each corresponding to one position of the bump or ramp on the propagation path. The data set identification card which follows each "DATA" card contains the values of ρ, $\Delta\rho_1$, $\Delta\rho_2$, and the hs heights. An additional identification card corresponding to the FASTMC input control card "BUMP" may be output.

The order of execution of the program is specified by a set of control cards. Each card is read and printed in its entirety, but only the first four columns are examined by the program. These control cards and their functions are described below.

> NAME signals that NAMELIST data follows. The NAMELIST is DATUM. The input variables are defined as follows:

RHO0   = ρ_0 in Mm - DEFAULT = 1.0

RHOL   = ρ_L in Mm - DEFAULT = 2.0

DRHO   = $\Delta\rho$ in Mm - DEFAULT = 0.1

DRHO1   = $\Delta\rho_1$ in Mm - DEFAULT = 0.1

DRHO2   = $\Delta\rho_2$ in Mm - DEFAULT = 20.0

RHOMAX = ρ_{max} in Mm - DEFAULT = 10.0

HPRIME = h' in km.   Maximum number of values is 7 - DEFAULT =
          87.0, 85.0, 83.0, 81.0, 79.0, 77.0, 75.0

ID   signals that the next card is to be read for identification of
        the output. The first 76 columns of this card are stored on a
        card with BUMP in its first 4 columns. This can be processed
        by FASTMC.

DATA   signals that data corresponding to one of the values of h'
        follow. These data are in the same format as required by
        FASTMC. They must be input in the order indicated by the
        HPRIMEs in the NAMELIST. The data for HPRIME(n) are stored in
        logical file 10 + n. The individual segments are stored in
        random access files, with each record representing a multiple
        of 5 km with a maximum distance of 40 Mm. Thus, data at 1.234
        Mm will be stored as 1.235 Mm in record number 248.

START   signals that all necessary inputs are complete and the genera-
        tion of the output data sets is to begin. The output is
        written to external file "OUTPUT" (logical file 2) in FASTMC
        card image format.

    The resulting data are used in FASTMC to obtain the variation of field
strength parametric in the position of the perturbation. Various programs
exist or could be written to display this output in other forms. Appendix B is
a listing of the program.

To illustrate the execution of BUMP with FASTMC, let us assume that a file named NLK exists. This file contains data elements for a variety of ionospheric profiles on the propagation path NLK to Brisbane, Australia. The electron density profile is exponential in height with a slope β and a reference height h' denoted by β/h'. The data elements in NLK are renamed BBHH, where BB and HH denote β and h', respectively. For example, the daytime ionosphere is taken to be 0.3/74 and the corresponding data element is named 3074. The nighttime ionosphere is taken to be 0.5/86. The variation in ionosphere from day to night is made in five equally spaced steps in β and h'. The output from BUMP is to go into a temporary file named OUTPUT, which is to be used in FASTMC to generate the resulting mode sums. The data deck to carry out these operations is shown in figure 6. The resulting plot is shown in figure 7. Note that the use of the "ID" control card in the input to the BUMP program results in additional identification of the curves via the "BUMP" control card in FASTMC. Also note that although β as well as h' varies, there is provision for identifying only the latter; the β variation must be indicated in the "ID" card.

```
@ASG,T OUTPUT.,F///1000
@XQT PGM.BUMP
ID
NLK TO BRISBANE, NIGHT (0.5/86) TO DAY (0.3/74)
NAME
 &DATUM RHO0=2., RHOL=5., DRHO=1., DRHO1=.150,
 RHOMAX=12.,
 HPRIME=86., 84., 82., 80., 78., 76., 74.,
 &END
@ADD NLK.5086
@ADD NLK.4784
@ADD NLK.4382
@ADD NLK.4080
@ADD NLK.3778
@ADD NLK.3376
@ADD NLK.3074
START
@XQT PGM.FASTMC
NAME
 &DATUM NPRINT=0, NRCURV=4, DMAX=12., SIZEX=6., AMPMIN=-10., SIZEY=4., &END
@ADD OUTPUT.
```

Figure 6.   Sample card deck for running BUMP and FASTMC.
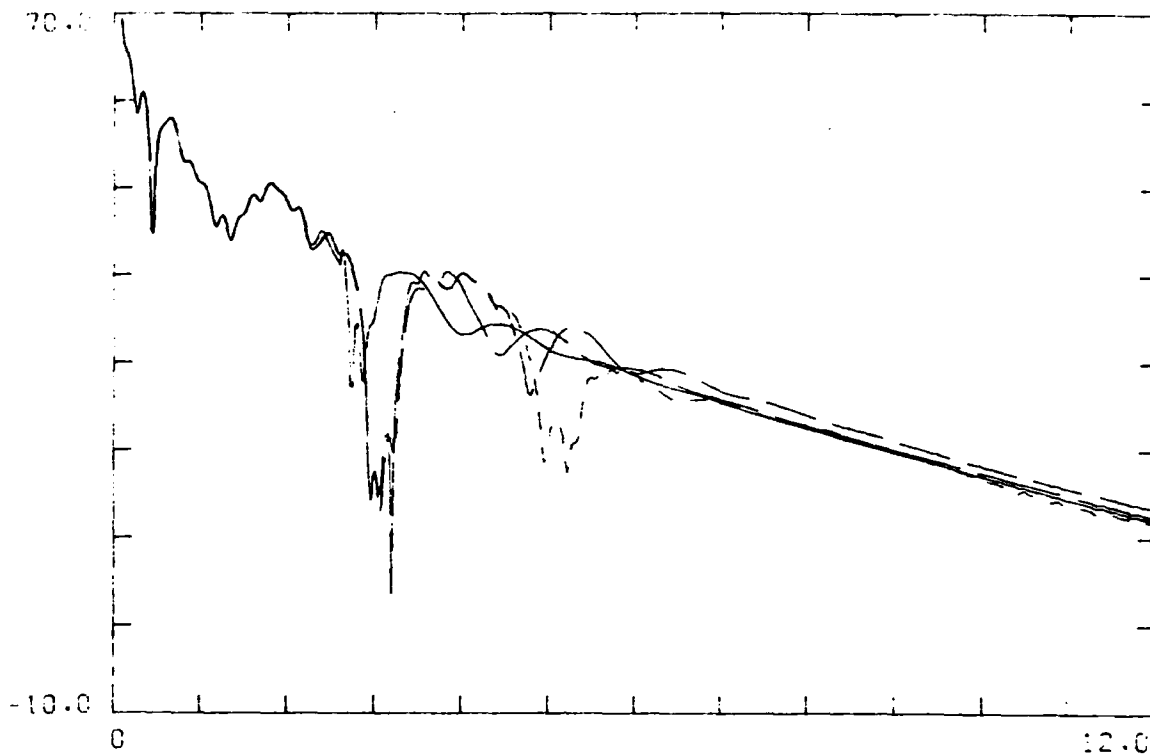
Figure 7.   Plots from data deck of figure 6.

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APPENDIX A:

FASTMC

```
C
C     APPROXIMATE MODE CONVERSION
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/MC INP1/XTRA(3,20),TERM(20),FREQ,OMEGA,WAVENO,MIK,
     $   ALPHA,AK,AK13,KA13,KA23,NGSQ,NTHSQ,CONST,ACONST,ECONST
      COMMON/MC STOR/A(20,20),FOFR(20),S(20),TP(20),XVAL,SIGMA,EPSR,
     $   NRMODE,NRSLAB
      COMMON/MC PLOT/DMIN,DMAX,XINC,AMPMIN,AMPMAX,AMPINC,PHSMIN,PHSMAX,
     $   PHSINC,IDPLOT(20),BUMPID(20),FRQ,POWER,ICOMP,INCL,THETA,
     $   TALT,RALT,IDENT,NRPTS,SIZEX,SIZEY,DATE,TIME,NRCURV,
     $   NRCRVS,NRPLTS,NAPLOT,NPPLOT,NPRINT
      COMMON SUMOUT(501)
      COMPLEX*16 TP,A,S,FOFR,XTRA,TERM,NGSQ,MI/(0.D0,-1.D0)/,MIK,
     $   TMP1,TMP2,TMP3,TMP4,RATIO(4)
C
      COMPLEX* 8 SUMOUT
      REAL*8 KA13,KA23,NTHSQ
      REAL*4 DMIN,DMAX,XINC,AMPMIN,AMPMAX,AMPINC,PHSMIN,PHSMAX,PHSINC,
     $   SIZEX,SIZEY,SIZEY1,SIZEX1,SIZEY1,FRQ,TPHT,TALT,RALT,
     $   INCL,THETA,TLONG,TLAT,RBEAR,POWER
      INTEGER TOTAPE
      CHARACTER*8 DATE,TIME
      CHARACTER*4 BCD,PLOTID,BUMPID
      CHARACTER   LABEL,IDENT
C
C  ICOMP          RECEIVED COMPONENT, =1 IS Z, =2 IS Y
C  INCL,THETA     ORIENTATION OF TRANSMITTER ANTENNA
C  TALT,RALT      TRANSMITTER,RECEIVER ALTITUDE IN KM
C  TOPHT          HEIGHT OF THE BOTTOM OF THE IONOSPHERE IN KM
C  DMIN,DMAX,XINC DISTANCE RANGE AND TIC INTERVAL IN MM
C  NAPLOT         AMPLITUDE PLOT FLAG
C  AMPMIN,AMPMAX,AMPMAX   AMPLITUDE PLOT RANGE AND TIC INTERVAL IN DB
C  NPPLOT         PHASE PLOT FLAG
C  PHSMIN,PHSMAX,PHSMAX   PHASE PLOT RANGE AND TIC INTERVAL IN DEGREES
C  SIZEX,SIZEY    AXIS LENGTHS IN INCHES
C  SIZEX1,SIZEY1  AXIS LENGTHS IN INCHES
C  NRCURV         NUMBER OF CURVES PER GRAPH, MAX OF 4
C  NRPTS          NUMBER OF POINTS TO OUTPUT, MAX OF 501
C  NPRINT         PRINTOUT FLAG
C  TOTAPE         OUTPUT FLAG, =1 WRITES TO LOGICAL UNIT 2
C  TLONG,TLAT     TRANSMITTER LOCATION IN DEGREES WEST,NORTH
C  RBEAR          PATH BEARING IN DEGREES EAST OF NORTH
C  POWER          RADIATED POWER IN KW
C
C  NOTE:  TLONG,TLAT,RBEAR ARE FOR TOTAPE=1 OUTPUT ONLY
C
      NAMELIST/DATUM/ICOMP,INCL,THETA,TALT,RALT,TOPHT,
     $   NAPLOT,NPPLOT,NPRINT,NRCURV,SIZEX,SIZEY,
     $   SIZEX1,SIZEY1,AMPMAX,AMPMIN,AMPINC,PHSMAX,PHSMIN,PHSINC,
     $   DMAX,DMIN,XINC,TLONG,TLAT,RBEAR,POWER,TOTAPE,NRPTS
C
      DIMENSION STORE(924),BCD(20),LABEL(2)
      EQUIVALENCE (STORE,A),(SIZEX,SIZEX1),(SIZEY,SIZEY1)
      DATA LABEL/'Z','Y'/
      DATA EPSLNO/8.85434D-12/,ALPHA/3.14D-04/,DTR/.01745329252D0/
      DATA ICOMP/1/,INCL,THETA/0.,0./,TALT,RALT/0.,0./,TOPHT/90.D0/
```

```fortran
      DATA TLONG,TLAT,RBEAR/0.,0.,0./,POWER/1./
      DATA DMIN,DMAX,XINC/0.,10.,.1./
      DATA AMPMIN,AMPMAX,AMPINC/10.,70.,10./
      DATA PHSMIN,PHSMAX,PHSINC/-360.,360.,90./
      DATA SIZEX,SIZEY/10.,6./
      DATA NRPTS/501/,NRCURV/1/,NAPLOT/1/,NPPLOT/0/,NPRINT/1/
      DATA NRCRVS,NRPLTS/0,0/,BUMPID/20*' '/
C
C     GET DATE AND TIME FOR PLOT IDENTIFICATION
      CALL ADATE(DATE,TIME)
      PRINT 1004,DATE,TIME
C
C     GET CONTROL CARD
10    PRINT 1001
      READ(5,1002,END=998) BCD
      PRINT 1003,BCD
      IF(BCD(1) .EQ. 'NAME') GO TO 12
      IF(BCD(1) .EQ. 'BUMP') GO TO 15
      IF(BCD(1) .EQ. 'DATA') GO TO 20
      IF(BCD(1) .EQ. 'STAR') GO TO 30
      GO TO 910
C
C     READ NAMELIST
12    READ DATUM
      PRINT DATUM
      IF(NRCURV .GT. 4) GO TO 913
      IF(SIZEY .GT. 9.) GO TO 918
      IF(NRPTS .GT. 501) GO TO 919
      GO TO 10
C
C     STORE BUMP ID CARD
15    DO 16 I=1,20
16    BUMPID(I)=BCD(I)
      GO TO 10
C
C     READ PROPAGATION PATH DATA
20    READ(5,1002,END=915) IDPLOT
      PRINT 1003,IDPLOT
      NRMODE=20
      NRSLAB=0
      RHO=-1.
      NTHSQ=1.+ALPHA*TOPHT
      REWIND 4
21    READ(5,1020,END=915) RR,FF,AA,CC,BB,SS,EE,TH
      IF(RR .NE. 40. .AND. SS .EQ. 0.) GO TO 21
      XVAL=RR*1000.
      IF(NRSLAB .GT. 0) WRITE(4) STORE
      IF(RR .EQ. 40.) GO TO 25
      NRSLAB=NRSLAB+1
      NM=0
      BB=BB*10000.
      PRINT 1022,NRSLAB,RR,FF,AA,CC,BB,SS,EE,TH
      IF(NPRINT .GE. 3) PRINT 1024
      IF(RR .GT. 0.) GO TO 22
      FRQ=FF
      FREQ=FF
      CONST=682.2408*DSQRT(FREQ*POWER)
```

```
114            OMEGA=6.28318530803*FREQ
115            WAVENO=20.958145D-3*FREQ
116            MIK=DCMPLX(0.D0,-WAVENO)
117            ACONST=-8.68GD3*WAVENO
118            ECONST=70.*DLOG10(35.*WAVENO)
119            AK=ALPHA/WAVENO
120            AK13=DEXP(DLOG(AK)/3.D0)
121            KA13=1.D0/AK13
122            KA23=KA13**2
123    22      IF(RHO .GT. RR) GO TO 912
124            RHO=RR
125            SIGMA=SS
126            EPSR=EE
127            NGSQ=DCMPLX(EPSR,-SIGMA/(OMEGA EPSLNO))
128            IF(TH .EQ. 0.) GO TO 23
129            NTHSQ=1.+ALPHA*TH
130    23      READ(5,1023,END=915) INDX1,TR1.TI1,ITRM1,TMP1,TMP2
131            IF(TR1 .EQ. 0.) GO TO 24
132            READ(5,1023,END=915) INDX2,TR2,TI2,ITRM2,TMP3,TMP4
133            IF(NM .EQ. 20) GO TO 233
134            IF(CDABS(TMP1) .EQ. 0.) GO TO 233
135            IF(TR1 .NE. TR2 .OR. TI1 .NE. TI2) GO TO 234
136            IF(ITRM1 .NE. ITRM2) GO TO 234
137            IF(ITRM1 .EQ. 0) GO TO 911
138            NM=NM+1
139            IF(NPRINT .LT. 3) GO TO 230
140            PRINT 1025,NM,INDX1,TR1,TI1,ITRM1,TMP1,TMP2,
141    $                INDX2,TR2,TI2,ITRM2,TMP3,TMP4
142    230     TP(NM)=DCMPLX(TR1,TI1)
143            S(NM)=CDSIN(TP(NM)*DTR)
144            RATIO(2,INDX1-1)=TMP1
145            RATIO(2,INDX1  )=TMP2
146            RATIO(2,INDX2-1)=TMP3
147            RATIO(2,INDX2  )=TMP4
148            TERM(NM)=RATIO(4)
149    C       GET EY/HY
150            IF(ITRM1 .EQ. 2) GO TO 231
151            FOFR(NM)=RATIO(3)/RATIO(1)
152            GO TO 232
153    231     FOFR(NM)=RATIO(2)/(RATIO(3)*RATIO(4))
154    232     IF(NRSLAB .GT. 1) GO TO 23
155    C
156    C       GET HY EXCITATION FACTOR AT THE TRANSMITTER
157            XTRA(1,NM)=-RATIO(1)*S(NM)
158            XTRA(2,NM)= RATIO(3)*RATIO(4)
159            XTRA(3,NM)= RATIO(1)
160            GO TO 23
161    233     IF(NPRINT .LT. 3) GO TO 23
162    234     PRINT 1026,   INDX1,TR1,TI1,ITRM1,TMP1,TMP2,
163    $                     INDX2,TR2,TI2,ITRM2,TMP3,TMP4
164            IF(TR1 .NE. TR2 .OR. TI1 .NE. TI2) GO TO 916
165            IF(ITRM1 .NE. ITRM2) GO TO 917
166            GO TO 23
167    24      IF(NPRINT .LT. 3) PRINT 1027,NM
168            NRMODE=NM
169    C
170    C       GET CONVERSION COEFFICIENTS
```

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```fortran
171           CALL MCSTEP
172           GO TO 21
173     25    IF(NRSLAB .LE. 1) GO TO 914
174     C
175     C     CALCULATE MODE SUM
176           IDENT=LABEL(ICOMP)
177     30    CALL MCFLDS
178     C
179           IF(TOTAPE .EQ. 0) GO TO 40
180     C     SAVE MODE SUM
181           WRITE(2) SUMOUT,FRQ,TLONG,TLAT,RBEAR,POWER,INCL,THETA,TALT,RALT,
182          $ DMIN,DMAX
183     C
184     C     MODE SUM OUTPUT
185     40    CALL MCPLTS
186           PRINT 1000
187           GO TO 10
188     C
189     C     ERROR EXITS
190     910   PRINT 1910
191           GO TO 998
192     911   PRINT 1911
193           GO TO 998
194     912   PRINT 1912
195           GO TO 998
196     913   PRINT 1913
197           GO TO 998
198     914   PRINT 1914
199           GO TO 998
200     915   PRINT 1915
201           GO TO 998
202     916   PRINT 1916
203           GO TO 998
204     917   PRINT 1917
205           GO TO 998
206     918   PRINT 1918
207           GO TO 998
208     919   PRINT 1919
209     C
210     998   PRINT 1998,NRCRVS,NRPLTS
211           IF(NRCRVS .EQ. 0) GO TO 999
212           CALL PLOT(0.,0.,999)
213     999   STOP
214     C
215     1000  FORMAT('1')
216     1001  FORMAT(' ')
217     1002  FORMAT(20A4)
218     1003  FORMAT(1X,20A4)
219     1004  FORMAT('0ADDITIONAL PLOT IDENTIFICATION: ',A8,1X,A8)
220     1020  FORMAT(1X,F7.0,3(2X,F8.0),2(2X,E10.0),2(2X,E5.0))
221     1022  FORMAT('0SLAB ',I2,' R',F7.3,' F',F8.4,' A',F8.3,' C',F8.3,' M',
222          $ F6.3,' S',1PE10.3,' E',0PF5.1,' T',F5.1)
223     1023  FORMAT(I1,2F9.0,I1,4E15.0)
224     1024  FORMAT(/11X,' M  ID     THETA')
225     1025  FORMAT(/11X,I2,3X,I1,0P2F10.5,I2,2(1X,1P2E16.8)/
226          $      16X,      I1,0P2F10.5,I2,2(1X,1P2E16.8))
227     1026  FORMAT(16X,      I1,0P2F10.5,I2,2(1X,1P2E16.8)/
```

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```
228    $                16X,            I1,0P2F10.5,I3,2(1X,1P2E16.8))
229 1027   FORMAT('+',80X,' MODES',I3)
230 1910   FORMAT('0*****   ERROR IN CONTROL CARD')
231 1911   FORMAT('0*****   THIS DATA DECK IS MISSING THE FOFR FLAG IN 20')
232 1912   FORMAT('0*****   XVALS OUT OF ORDER')
233 1913   FORMAT('0*****   NRCURV MUST BE LESS THAN OR EQUAL TO 4')
234 1914   FORMAT('0*****   NUMBER OF SLABS LESS THAN 2')
235 1915   FORMAT('0*****   END OF DATA SET ON UNIT 5')
236 1916   FORMAT('0*****   ERROR IN DATA SEQUENCE')
237 1917   FORMAT('0*****   ITERM FLAG INCONSISTENT')
238 1918   FORMAT('0*****   SIZEY MUST BE LESS THAN OR EQUAL TO 9')
239 1919   FORMAT('0*****   NRPTS MUST BE LESS THAN OR EQUAL TO 501')
240 1998   FORMAT('0END OF JOB, ',I2,' CURVES PLOTTED IN ',I2,' GRAPHS')
241        END
```

```fortran
      SUBROUTINE MCSTEP                                                      1
C                                                                            2
C     SUBROUTINE TO CALCULATE MODE CONVERSION COEFFICIENTS                   3
C                                                                            4
      IMPLICIT REAL*8 (A-H,O-Z)                                              5
      COMMON/MC INPT/XTRA(3,20),TERM(20),FREQ,OMEGA,WAVENO,MIK,              6
     $  ALPHA,AK,AK13,KA13,KA23,NGSQ,NTHSQ,SKIP(3)                           7
      COMMON/MC STOR/A(20,20),FOFR(20),S(20),SKIP1(43),NRMODE,NRSLAB         8
      COMMON/MC PLOT/ISKIP(69),NPRINT                                        9
      COMMON NORM(20,20),CAPI(20),AN(20),PS(20),                           10
     $  EXO(20), EYO(20), HXO(20),                                         11
     $  PEXO(20),PEYO(20),PHXO(20),                                        12
     $  EXT(20), EYT(20), HXT(20), HYT(20), TAU(20),                       13
     $  PEXT(20),PEYT(20),PHXT(20),PHYT(20),PTAU(20)                       14
      COMPLEX*16 THETAP,FOFR,XTRA,TERM,ANS,A,S,SSQ,SJ,SM,TJ,TM,PS,         15
     $  QO,H1O,H2O,H1PRMO,H2PRMO,QT,H1T,H2T,H1PRMT,H2PRMT,                 16
     $  A1,A2,A3,A4,MIK,EYOJ,EYOM,                                        17
     $  FPRP,DFPRP,FPRL,DFPRL,MULT,ARGT,ARGO,ARGG,NORM,CAPI,              18
     $  EXO, EXT, EYO, EYT, HXO, HXT, HYT, NGSQ, TAU,                     19
     $  PEXO,PEXT,PEYO,PEYT,PHXO,PHXT,PHYT,PNGSQ,PTAU,                    20
     $  ONE/(1.D0,0.D0)/,ZERO/(0.D0,0.D0)/,                               21
     $  W/(0.D0,-1.4574954400)/                                           22
      REAL*8 KA13,KA23,NTHSQ                                               23
      INTEGER PNMODE                                                       24
C                                                                          25
      DO 90 M=1,NRMODE                                                     26
      THETAP=M                                                             27
      SSQ=S(M)**2                                                          28
      TM=CDSQRT(NGSQ-SSQ)                                                  29
      TAU(M)=TM                                                            30
      A2=DCMPLX(0.D0,KA13)*TM                                             31
      A1=A2/NGSQ                                                           32
      QO=KA23*(ONE-SSQ)                                                    33
      CALL MDHNKL(QO,H1O,H2O,H1PRMO,H2PRMO,THETAP,'MCO  ')               34
      EXO(M)=-TM/NGSQ                                                     35
      EYO(M)=FOFR(M)                                                      36
      HXO(M)=TM*FOFR(M)                                                   37
      HYO     =ONE                                                        38
      QT=KA23*(NTHSQ-SSQ)                                                 39
      CALL MDHNKL(QT,H1T,H2T,H1PRMT,H2PRMT,THETAP,'MCT  ')               40
      A3=H1T   *H2O  -H1O  *H2T                                          41
      A4=H1T   *H2PRMO-H1PRMO*H2T                                        42
      FPRL=A4-A1*A3                                                       43
      FPRP=A4-A2*A3                                                       44
      A3=H1PRMT*H2O   -H1O  *H2PRMT                                      45
      A4=H1PRMT*H2PRMO-H1PRMO*H2PRMT                                     46
      DFPRL=A4-A1*A3                                                      47
      DFPRP=A4-A2*A3                                                      48
      EXT(M)=DCMPLX(0.D0,AK13)*DFPRL/W                                   49
      EYT(M)=EYO(M)*FPRP/W                                               50
      HXT(M)=DCMPLX(0.D0,-AK13)*EYO(M)*DFPRP/W                           51
      HYT(M)=FPRL/W                                                      52
   90 CONTINUE                                                           53
C                                                                        54
C                                                                        55
      IF(NRSLAB .EQ. 1) GO TO 30                                         56
```

```fortran
C        INTEGRALS IN CURRENT SLAB
         IF(NPRINT .GT. 2) PRINT 906,NRSLAB
         DO 140 M=1,NRMODE
         SM=S(M)
         TM=TAU(M)
         EYOM=EYO(M)
         IF(NPRINT .GT. 2) PRINT 902
         DO 140 J=1,NRMODE
         IF(J .EQ. M) GO TO 120
         SJ=S(J)
         TJ=TAU(J)
         EYOJ=EYO(J)
         MULT=ONE/(MIK*(SJ-SM))
         ARGT=TERM(M)*(EYT(M)*HXT(J)-EYT(J)*HXT(M))
     $          -       (HYT(M)*EXT(J)-HYT(J)*EXT(M))
         ARGO=TERM(M)*(EYO(M)*HXO(J)-EYO(J)*HXO(M))
     $          -       (      EXO(J)-       EXO(M))
         ARGG=(ONE+EYOJ*EYOM)*(SJ+SM)/(MIK*(TJ+TM))
         GO TO 130
C        MODES ARE EQUAL
120      MULT=SM*(2.D0/ALPHA)
         SSQ=SM**2
         QO=ONE-SSQ
         QT=NTHSQ-SSQ
         ARGT=TERM(M)*(QT*EYT(M)**2-HXT(M)**2)+(QT*HYT(M)**2-EXT(M)**2)
         ARGO=TERM(M)*(QO*EYO(M)**2-HXO(M)**2)+(QO        -EXO(M)**2)
         ARGG=(ONE+EYOM**2)*SM/(MIK*TM)
         NORM(J,M)=MULT*(ARGT-ARGO)-ARGG
         IF(NPRINT .GT. 2) PRINT 908,M,J,NORM(J,M)
130      CONTINUE
140      CONTINUE
C
         IF(NTHSQ .EQ. PNTHSQ) GO TO 300
C        PREVIOUS SLAB HAD DIFFERENT TOPHT, MUST RECOMPUTE FIELDS
         DO 200 J=1,PNMODE
         THETAP=J
         SSQ=PS(J)**2
         A2=DCMPLX(0.D0,KA13)*PTAU(J)
         A1=A2/PNGSQ
         QO=KA23*(ONE-SSQ)
         CALL MDHNKL(QO,H1O,H2O,H1PRMO,H2PRMO,THETAP,'MCPO')
         QT=KA23*(NTHSQ-SSQ)
         CALL MDHNKL(QT,H1T,H2T,H1PRMT,H2PRMT,THETAP,'MCPT')
         A3=H1T    *H2O    -H1O    *H2T
         A4=H1T    *H2PRMO-H1PRMO*H2T
         FPRL=A4-A1*A3
         FPRP=A4-A2*A3
         A3=H1PRMT*H2O    -H1O    *H2PRMT
         A4=H1PRMT*H2PRMO-H1PRMO*H2PRMT
         DFPRL=A4-A1*A3
         DFPRP=A4-A2*A3
         PEXT(J)=DCMPLX(0.D0,AK13)*DFPRL/W
         PEYT(J)=PEYO(J)*FPRP/W
         PHXT(J)=DCMPLX(0.D0,-AK13)*PEYO(J)*DFPRP/W
         PHYT(J)=FPRL/W
200      CONTINUE
C
C        INTEGRALS ACROSS SLAB BOUNDARY
```

```
114  300     INIT=0
115          IF(NPRINT .GT. 1) PRINT 900,NRSLAB
116          DO 500 J=1,PNMODE
117          SJ=PS(J)
118          SJR=SJ
119          SJI=SJ*(0.D0,-1.D0)
120          TJ=PTAU(J)
121          EYOJ=PEYO(J)
122          IF(NPRINT .GT. 1) PRINT 902
123          DO 400 M=1,NRMODE
124          SM=S(M)
125          SMR=SM
126          SMI=SM*(0.D0,-1.D0)
127          TM=TAU(M)
128          EYOM=EYO(M)
129          IF(SMR .EQ. SJR .AND. SMI .EQ. SJI) GO TO 350
130          MULT=ONE/(MIK*(SJ-SM))
131          ARGT=TERM(M)*(EYT(M)*PHXT(J)-PEYT(J)*HXT(M))
132     $            (HYT(M)*PEXT(J)-PHYT(J)*EXT(M))
133          ARGO=TERM(M)*(EYO(M)*PHXO(J)-PEYO(J)*HXO(M))
134     $            (               PEXO(J)-          EXO(M))
135          ARGG=(ONE+EYOJ*EYOM)*(SJ+SM)/(MIK*(TJ+TM))
136          GO TO 390
137  C       MODES ARE EQUAL
138  350     MULT=SM*(2.D0/ALPHA)
139          SSQ=SM**2
140          QO=ONE-SSQ
141          QT=NTHSQ-SSQ
142          ARGT=TERM(M)*(QT*EYT(M)**2-HXT(M)**2)+(QT*HYT(M)**2-EXT(M)**2)
143          ARGO=TERM(M)*(QO*EYO(M)**2-HXO(M)**2)+(QO           -EXO(M)**2)
144          ARGG=(ONE+EYOM**2)*SM/(MIK*TM)
145          CAPI(M)=MULT*(ARGT-ARGO)-ARGG
146  390     IF(NPRINT .GT. 2) PRINT 910,M,J,CAPI(M)
147  400     CONTINUE
148  C
149  C       CALCULATE CONVERSION COEFFICIENTS FOR HY
150          CALL CLINEQ(NORM,CAPI,ANS,NRMODE,20,INIT,ERR)
151          INIT=1
152          IF(NPRINT .GT. 2) PRINT 902
153          DO 430 M=1,NRMODE
154          A(J,M)=ANS(M)
155          IF(NPRINT .LT. 2) GO TO 430
156          AR=ANS(M)
157          AI=ANS(M)*(0.D0,-1.D0)
158          DB=10.*DLOG10(AR*AR+AI*AI)
159          ANG=DATAN2(AI,AR)*57.295779SD0
160          PRINT 903,M,J,AR,AI,DB,ANG
161  430     CONTINUE
162  500     CONTINUE
163          GO TO 50
164  C
165  C       FIRST SLAB
166  30      DO 32 M=1,NRMODE
167          DO 32 J=1,NRMODE
168          IF(J .EQ. M) GO TO 31
169          A(J,M)=ZERO
170          GO TO 32
```

```
171    31    A(J,M) ONE
172    32    CONTINUE
173    C
174    C     SAVE DATA FOR NEXT SLAB
175    50    PXVAL=XVAL
176          PNGSQ=NGSQ
177          PNTHSQ=NTHSQ
178          PNMODE=NRMODE
179          DO 60 J=1,PNMODE
180          PS(J)=S(J)
181          PTAU(J)=TAU(J)
182          PEXO(J)=EXO(J)
183          PEYO(J)=EYO(J)
184          PHXO(J)=HXO(J)
185          PEXT(J)=EXT(J)
186          PEYT(J)=EYT(J)
187          PHXT(J)=HXT(J)
188    60    PHYT(J)=HYT(J)
189          RETURN
190    900   FORMAT(' OCONVERSION COEFFICIENTS FOR SLAB ',I2)
191    901   FORMAT(' A(',I2,',',I2,')=',1P2E15.5,5X,
192      $          '20*LOG10(A)=',0P2F10.3)
193    902   FORMAT(' ')
194    903   FORMAT(' T(',I2,',',I2,')=',1P2E15.5,5X,
195      $          '20*LOG10(T)=',0P2F10.3)
196    906   FORMAT(' OINTEGRALS IN SLAB',I3)
197    908   FORMAT(' NORM(',    I2,',',I2,') =',1P2E15.6)
198    910   FORMAT(' CAPI(',    I2,',',I2,) =',1P2E15.6)
199          END
```

```
 1            SUBROUTINE MCFLDS
 2      C
 3      C     SUBROUTINE TO CALCULATE MODE SUM
 4      C
 5            IMPLICIT REAL*8 (A-H,O-Z)
 6            COMMON/MC INPT/XTRA(3,20),TERM(20),FREQ,OMEGA,WAVENO,MIK,
 7           $   ALPHA,SKIP(7),CONST,ACONST,ECONST
 8            COMMON/MC STOR/A(20,20),FOFR(20),S(20),TP(20),XVAL,SIGMA,EPSR,
 9           $   NRMODE,NRSLAB
10            COMMON/MC FLDI/DMIN,DMAX,ISKIP1(7),IDPLOT(20),ISKIP2(22),RCOMP,
11           $   INCL,THETA,TALT,RALT,LABEL,NRPTS,ISKIP3(11),NPRINT
12            COMMON SUMOUT(501),R(501),DB(501),ANG(501),SOLNA(20),SOLNB(20),
13           $   HGT(3,20),HGR(3,20)
14            DIMENSION STORE(924),T(3)
15            EQUIVALENCE (STORE,A)
16            COMPLEX*16 A,S,TP,FOFR,XTRA,TERM,SOLNA,SOLNB,HGT,HGR,
17           $   TA,TB,MI/(0.D0,-1.D0),ONE/(1.D0,0.D0)/,MIK,MIKX
18            COMPLEX* 8 SUMOUT
19            REAL*8 NTHSQ
20            REAL*4 R,DB,ANG,INCL,THETA,TALT,RALT,DMIN,DMAX
21            INTEGER RCOMP,TCOMP,PNMODE
22      C
23            REWIND 4
24            DELRHO=(DMAX-DMIN)*1000./FLOAT(NRPTS-1)
25      C
26      C     ANTENNA ORIENTATION FACTORS
27            T(1)=COS(INCL*.01745329252)
28            T(2)=SIN(INCL*.01745329252)*SIN(THETA*.01745329252)
29            T(3)=SIN(INCL*.01745329252)*COS(THETA*.01745329252)
30            IF(NPRINT .EQ. 0) GO TO 200
31            PRINT 1030,IDPLOT,LABEL,INCL,THETA,TALT,RALT
32      C
33      C     M IS SLAB INDEX
34      C     X IS DISTANCE OF PREVIOUS SLAB FROM THE TRANSMITTER
35      C     XVAL IS DISTANCE OF NEXT SLAB
36      C
37  200       PHSI=0.
38            CYCLE=0.
39            IF(DMIN .NE. 0.) GO TO 210
40            R(1)=0.
41            DB(1)=CONST*80.D0
42            ANG(1)=0.
43            SUMOUT(1)=DB(1)
44            ISUB=1
45            RHO=DELRHO
46            GO TO 220
47  210       ISUB=0
48            RHO=DMIN*1000.
49  220       M=1
50            X=0.D0
51            GO TO 660
52      C
53      C     END OF CURRENT SLAB
54  600       MIKX=MIK*(XVAL-X)
55            PNMODE=NRMODE
56            DO 610 J=1,PNMODE
```

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```
C       GET EXCITATION FACTORS AT END OF SLAB
        SOLN B(J)=SOLN B(J)*CDEXP(MIKX*(S(J)-ONE))
610     TERM   (J)=SOLN B(J)
        M=M+1
        X=XVAL
C
C       GET DATA FOR NEXT SLAB
660     READ(4) STORE
        MIKX=MIK*XVAL
        CALL HIGAIN(1,FREQ,SIGMA,EPSR,ALPHA,NRMODE,TP,DBLE(RALT),HGR)
C
        IF(M .EQ. 1) GO TO 685
C
        DO 681 J=1,NRMODE
C       HY EXCITATION FACTOR
        TA=(0.D0,0.D0)
        DO 665 K=1,PNMODE
665     TA=TA+TERM(K)*A(K,J)
        SOLN B(J)=TA
        IF(RCOMP .EQ. 2) GO TO 671
C       EZ EXCITATION FACTOR
        TA=-S(J)*TA
        GO TO 680
C       EY EXCITATION FACTOR
671     TA=FOFR(J)*TA
680     SOLN A(J)=TA+HGR(RCOMP,J)
681     CONTINUE
        GO TO 690
C
C       FIRST SLAB
685     CALL HIGAIN(1,FREQ,SIGMA,EPSR,ALPHA,NRMODE,TP,DBLE(TALT),HGT)
        DO 689 J=1,NRMODE
C       HY EXCITATION FACTOR
        TA=(0.D0,0.D0)
        DO 686 TCOMP=1,3
686     TA=TA+XTRA(TCOMP,J)*HGT(TCOMP,J)*T(TCOMP)
        SOLN B(J)=TA
        IF(RCOMP .EQ. 2) GO TO 687
C       EZ EXCITATION FACTOR
        TA=-S(J)*TA
        GO TO 688
C       EY EXCITATION FACTOR
687     TA=FOFR(J)*TA
688     SOLN A(J)=TA+HGR(RCOMP,J)
689     CONTINUE
C
690     DIST=X/1000.D0
        IF(NPRINT .EQ. 0) GO TO 720
C
C       PRINT MODE CONSTANTS TABLE
        PRINT 1031,M,DIST
        DO 692 J=1,NRMODE
        SR=S(J)
        SI=S(J)*MI
        ATTEN=ACONST*SI
        VOVERC=1.D0/SR
        WR=SOLN A(J)
```

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```
114      W1=SOLN A(J),M1
115      WM1=ECUNST+10.D0*DLOG10(WR**2+W1**2)
116      WA1=DATAN2(WI,WR)-1.5707963267949D0
117      PRINT 1032,J,ATTEN,VOVERC,WM1,WA1
118  692 CONTINUE
119  C
120  C   GET SUM OF MODES
121  720 IF(RHO .GT. XVAL) GO TO 600
122      TA=0.D0
123      MIKX=MIK*(RHO-X)
124      DO 730 K=1,NRMODE
125      TB=SOLN A(K)*CDEXP(MIKX*(S(K)-UNE))
126  730 TA=TA+TB
127      TA=TA*CONST/DSQRT(DABS(DSIN(RHO/6.366D3)))
128      TDBR=TA
129      TDBI=TA*M1
130      PHS2=DATAN2(TDBI,TDBR)*57.2957/95D0
131      IF(DABS(PHS1-PHS2) .LT. 180.D0) GO TO 750
132      IF(PHS1 .GT. PHS2) GO TO 740
133      CYCLE=CYCLE-360.D0
134      GO TO 750
135  740 CYCLE=CYCLE+360.D0
136  750 PHS1=PHS2
137      ISUB=ISUB+1
138      R(ISUB)=RHO/1000.D0
139      DB(ISUB)=10.*DLOG10(TDBR**2+TDBI**2)
140      ANG(ISUB)=PHS2+CYCLE
141      SUMOUT(ISUB)=TA
142      RHO=RHO+DELRHO
143      IF(ISUB .LT. NRPTS) GO TO 720
144      RETURN
145  C
146 1030 FORMAT('1',20A4/1X,A1,' COMPONENT   INCL =',F4.0,'   RALT =',F5.1/
147     $        THETA =',F5.0,'   TALT =',F5.1,'   REL EXC 1')
148     $ 'OSLAB   RHO     MODE   ATTEN    V/C
149 1031 FORMAT(1X,I3,F9.3)
150 1032 FORMAT(13X,I3,F8.3,F9.5.2(F10.3,F7.3))
151      END
```

```
         SUBROUTINE MCPLTS
C
C     SUBROUTINE TO PLOT MODE SUM AMPLITUDE AND RELATIVE PHASE
C
         COMMON/MC PLOT/DMIN,DMAX,XINC,AMPMIN,AMPMAX,AMPINC,PHSMIN,PHSMAX,
     $      PHSINC,ID(20),BUMPID(20),FREQ,POWER,ICOMP,INCL,THETA,TALT,
     $      RALT,LABEL,NRPTS,SIZEX,SIZEY,DATIME(4),NRCURV,NRCRVS,
     $      NRPLTS,NAPLOT,NPPLOT,NPRINT
         COMMON SKIP1(1002),D(501),OUT1(501),OUT2(501),UP(501),
     $      XL(2),YL(2),UL(2)
         REAL INCL
         LOGICAL UP,UL
         DATA LINE/0/,BLANK/'      '/
C
         IR=-1
         LINE=LINE+1
         KK=MOD(LINE,NRCURV)
         IF(KK .EQ. 0) KK=NRCURV
         OFFSET=AMAX1(10.5,SIZEX+2.5)
C
         IF(NPRINT .EQ. 0) GO TO 20
         PRINT 1000,ID,LABEL,INCL,THETA,TALT,RALT
         NF=FLOAT(NRPTS)/4.+.99
         DO 11 I1=1,NF
11       PRINT 1011,(D(I2),OUT1(I2),OUT2(I2),I2=I1,NRPTS,NF)
C
20       IF(NAPLOT .EQ. 0) GO TO 30
         IR=IR+1
         DO 23 I=1,NRPTS
         UP(I)=.FALSE.
         IF(OUT1(I) .LE. AMPMAX) GO TO 21
         OUT1(I)=AMPMAX
         GO TO 22
21       IF(OUT1(I) .GE. AMPMIN) GO TO 23
         OUT1(I)=AMPMIN
22       UP(I)=.TRUE.
23       CONTINUE
         NPLOT=1
         YMAX=AMPMAX
         YMIN=AMPMIN
         YINC=AMPINC
         GO TO 100
C
30       IF(NPPLOT .EQ. 0) GO TO 120
         IR=IR+1
         FK=0.
         DO 37 I=1,NRPTS
         OUT1(I)=OUT2(I)+FK
         UP(I)=.FALSE.
         IF(OUT1(I) .LT. PHSMIN) GO TO 32
         IF(OUT1(I) .GT. PHSMAX) GO TO 34
         GO TO 37
32       IF(PHSMIN .EQ. 0.) GO TO 33
         FK=FK-PHSMIN
         OUT1(I)=OUT1(I)-PHSMIN
         GO TO 36
```

```
57   33   FK=FK+PHSMAX
58        OUT1(I)=OUT1(I)+PHSMAX
59        GO TO 36
60   34   IF(PHSMAX .EQ. 0.) GO TO 35
61        FK=FK-PHSMAX
62        OUT1(I)=OUT1(I)-PHSMAX
63        GO TO 36
64   35   FK=FK+PHSMIN
65        OUT1(I)=OUT1(I)+PHSMIN
66   36   UP(I)=.TRUE.
67        GO TO 31
68   37   CONTINUE
69        NPLOT=2
70        YMAX=PHSMAX
71        YMIN=PHSMIN
72        YINC=PHSINC
73   C
74   100  NRCRVS=NRCRVS+1
75        IF(KK .GT. 1) GO TO 109
76        IF(IR .GT. 0) GO TO 101
77        CALL SYMBOL(.1,0.,.1,DATIME,90.,16)
78        CALL PLOT(1.5,1.7,-3)
79        GO TO 102
80   101  CALL PLOT(OFFSET,0.,-3)
81   102  NRPLTS=NRPLTS+1
82        XP=0.
83        YP=-.4
84        CALL SYMBOL(XP,YP,.1,4HFMC ,0.,4)
85        XP=XP+0.5
86        CALL NUMBER(XP,YP,.1,FLOAT(NRPLTS),0.,-1)
87        XP=0.
88        YP=YP-.15
89        CALL SYMBOL(XP,YP,.1,LABEL,0.,1)
90        XP=XP+0.2
91        CALL SYMBOL(XP,YP,.1,9HCOMPONENT,0.,9)
92        XP=XP+1.0
93        IF(NPLOT .EQ. 2) GO TO 103
94        CALL SYMBOL(XP,YP,.1,9HAMPLITUDE,0.,9)
95        XP=XP+1.1
96        CALL SYMBOL(XP,YP,.1,7HPOWER =,0.,7)
97        XP=XP+0.8
98        CALL NUMBER(XP,YP,.1, POWER ,0.,0)
99        XP=XP+0.6
100       GO TO 104
101  103  CALL SYMBOL(XP,YP,.1,14HRELATIVE PHASE,0.,14)
102       XP=XP+1.5
103  104  CALL SYMBOL(XP,YP,.1,6HFREQ = ,0.,6)
104       XP=XP+0.7
105       CALL NUMBER(XP,YP,.1,FREQ,0.,3)
106       XP=0.
107       YP=YP-.15
108       CALL SYMBOL(XP,YP,.1,6HINCL =,0.,6)
109       XP=XP+0.7
110       CALL NUMBER(XP,YP,.1, INCL ,0.,0)
111       XP=XP+0.6
112       CALL SYMBOL(XP,YP,.1,7HTHETA =,0.,7)
113       XP=XP+0.8
```

```
114          CALL NUMBER(XP,YP,.1, THETA    ,0.,0)
115          XP=XP+0.8
116          CALL SYMBOL(XP,YP,.1,6HTALT = ,0.,6)
117          XP=XP+0.7
118          CALL NUMBER(XP,YP,.1,TALT,0.,1)
119          XP=XP+0.7
120          CALL SYMBOL(XP,YP,.1,6HRALT = ,0.,6)
121          XP=XP+0.7
122          CALL NUMBER(XP,YP,.1,RALT,0.,1)
123          IF(BUMPID(1) .EQ. BLANK) GO TO 108
124          XP=0.
125          YP=YP-.15
126          CALL SYMBOL(XP,YP,.1,BUMPID(1),0.,4)
127          XP=XP+0.5
128          CALL SYMBOL(XP,YP,.1,BUMPID(2),0.,72)
108  129     CALL BORDER(SIZEX,DMIN,DMAX,XINC,1,SIZEY,YMIN,YMAX,YINC,1)
130          GO TO 110
109  131     IF(IR .EQ. 0) GO TO 110
132          CALL PLOT(OFFSET,0.,-3)
   C 133
110  134     XP=0.
135          YP=-.85-.15*KK
136          XL(1)=-1.1
137          XL(2)=-0.1
138          YL(1)=YP
139          YL(2)=YP
140          UL(1)=.FALSE.
141          UL(2)=.FALSE.
142          CALL CURVE(XL,YL,UL,2,0.,0.,1.,1.,KK)
143          CALL SYMBOL(XP,YP,.1,ID,0.,80)
144          SCALEX=(DMAX-DMIN)/SIZEX
145          SCALEY=(YMAX-YMIN)/SIZEY
146          CALL CURVE(D,OUT1,UP,NRPTS,DMIN,YMIN,SCALEX,SCALEY,KK)
147          IF(NPLOT .EQ. 1) GO TO 30
   C 148
120  149     IF(IR .LT. 0) GO TO 999
150          IF(KK .EQ. NRCURV) GO TO 121
151          IF(IR .EQ. 0) GO TO 999
152          CALL PLOT(-OFFSET,0.,-3)
153          GO TO 999
121  154     CALL PLOT(0.,0.,-4)
155          LINE=0
156          RETURN
999  157
   C
1000 158     FORMAT('1',20A4/1X,A1,' COMPONENT    INCL =',F4.0,'  RALT =',F5.1/
159         $       THETA =',F5.0,'  TALT =',F5.1,'
160         $     4(' DIST    AMPLITUDE    PHASE',7X))
1011 161     FORMAT(4(F7.3,2F10.4,5X))
162          END
```

```
1    SUBROUTINE BORDER(XLNG,XMIN,XMAX,XINC,NX,YLNG,YMIN,YMAX,YINC,NY)
2    DIMENSION XINC(NX),YINC(NY)
3    LOGICAL FY,FX
4    FX=.FALSE.
5    FY=.FALSE.
6    IF(NX .EQ. 1) FX=.TRUE.
7    IF(NY .EQ. 1) FY=.TRUE.
8    XT=XLNG-.1
9    YT=YLNG-.1
10   XSCALE=XLNG/(XMAX-XMIN)
11   YSCALE=YLNG/(YMAX-YMIN)
12   YM=ABS(YMIN)
13   YLN=-.4
14   IF(YM .GE. 10.) YLN=YLN-.1
15   IF(YM .GE. 100.) YLN=YLN-.1
16   IF(YM .GE. 1000.) YLN=YLN-.1
17   IF(YMIN .LT. 0.) YLN=YLN-.1
18   YM=ABS(YMAX)
19   YLM=-.4
20   IF(YM .GE. 10.) YLM=YLM-.1
21   IF(YM .GE. 100.) YLM=YLM-.1
22   IF(YM .GE. 1000.) YLM=YLM-.1
23   IF(YMAX .LT. 0.) YLM=YLM-.1
24   XM=ABS(XMAX)
25   XLM=-.3
26   IF(XM .GE. 10.) XLM=XLM-.1
27   IF(XM .GE. 100.) XLM=XLM-.1
28   IF(XM .GE. 1000.) XLM=XLM-.1
29   IF(XMAX .LT. 0.) XLM=XLM-.1
30   IF(FX) DX=XINC(1)
31   IF(FY) DY=YINC(1)
32   IY=1
33   YL=0.
34   CALL NUMBER(YLN,0.,.1,YMIN,0.,1)
35 10   CALL PLOT(0.,0.,3)
36   IF(FY) GO TO 110
37   YP=(YINC(IY)-YMIN)*YSCALE
38   GO TO 111
39 110   YL=YL+DY
40   YP=YL*YSCALE
41 111   IF(YP .LT.   0.) GO TO 99
42   IF(YP .GE. YLNG) GO TO 11
43   CALL PLOT(0.,YP,2)
44   CALL PLOT(.1,YP,2)
45   CALL PLOT(0.,YP,2)
46   IF(FY) GO TO 110
47   IY=IY+1
48   IF(IY .LE. NY) GO TO 10
49 11   CALL PLOT(0.,YLNG,2)
50   CALL NUMBER(YLM,YLNG-.1,.1,YMAX,0.,1)
51   CALL PLOT(0.,YLNG,3)
52   IX=1
53   XL=0.
54   IF(FX) GO TO 112
55 12   XP=(XINC(IX)-XMIN)*XSCALE
56   GO TO 120
```

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```fortran
         XL=XL+DX
         XP=XL*XSCALE
112      IF(XP .LT.  0.) GO TO 99
120      IF(XP .GE. XLNG) GO TO 13
         CALL PLOT(XP,YLNG,2)
         CALL PLOT(XP,YT,2)
         CALL PLOT(XP,YLNG,2)
         IF(FX) GO TO 112
         IX=IX+1
         IF(IX .LE. NX) GO TO 12
13       CALL PLOT(XLNG,YLNG,2)
         IF(FY) GO TO 130
113      IY=IY-1
         IF(IY .LE. 0) GO TO 15
         YP=(YINC(IY)-YMIN)*YSCALE
         GO TO 14
130      YL=YL-DY
         YP=YL*YSCALE
         IF(YP .LE. 0.) GO TO 15
14       CALL PLOT(XLNG,YP,2)
         CALL PLOT(XT,YP,2)
         CALL PLOT(XLNG,YP,2)
         IF(FY) GO TO 130
         GO TO 113
15       CALL PLOT(XLNG,0.,2)
         CALL NUMBER(XLNG+XLM,-.2,.1,XMAX,0.,1)
         CALL PLOT(XLNG,0.,3)
         IF(FX) GO TO 150
115      IX=IX-1
         IF(IX .LE. 0) GO TO 17
         XP=(XINC(IX)-XMIN)*XSCALE
         GO TO 16
150      XL=XL-DX
         XP=XL*XSCALE
         IF(XP .LE. 0.) GO TO 17
16       CALL PLOT(XP,0.,2)
         CALL PLOT(XP,.1,2)
         CALL PLOT(XP,0.,2)
         IF(FX) GO TO 150
         GO TO 115
17       CALL PLOT(0.,0.,2)
         CALL NUMBER(0.,-.2,.1,XMIN,0.,1)
         RETURN
99       PRINT 100,XLNG,XMIN,XMAX,XINC(1),NX,YLNG,YMIN,YMAX,YINC(1),NY
100      FORMAT('0*** ERROR IN BORDER:  XLNG, XMIN, XMAX, XINC(1), NX =',
     $   1P4E15.5,I5/24X,'YLNG, YMIN, YMAX, YINC(1), NY =',1P4E15.5,
     $   I5/'0***')
         CALL PLOT(0.,0.,999)
         STOP
         END
```

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```fortran
      SUBROUTINE CURVE(X,Y,UP,NRPTS,XMIN,YMIN,XINC,YINC,LINE)
C
C  X,Y,UP MUST BE DIMENSIONED AT LEAST NRPTS
C  XMIN,YMIN ARE X,Y ORIGIN IN USER UNITS
C  XINC,YINC ARE X,Y SCALES IN USER UNITS PER INCH
C
C  LINE=1:  SOLID
C       2:  LONG DASH
C       3:  MEDIUM DASH
C       4:  SHORT DASH
C       5:  DOTTED
C       6:  SHORT + LONG DASH
C       7:  SHORT + SHORT + LONG DASH
C
C
      LOGICAL UP,UP1,UP2
      DIMENSION IPEN(10),JDC(7),X(NRPTS),Y(NRPTS),UP(NRPTS)
      DATA IPEN/3,2,3,2,3,2,2,2,2,2/,JDC/18, 61, 56, 54, 52, 11, 36/
      DATA DELR/.1/
C
      IF(LINE) 1,2,3
    1 KK=MOD(LINE,7)+7
      GO TO 4
    2 KK=0
      GO TO 4
    3 KK=MOD(LINE,7)
    4 KK=KK+1
      JD=JDC(KK)/10
      JC=JDC(KK)-10*JD
C
      J=1
      IP=2
      IF(KK .EQ. 6) IP=3
      DR=0.
      RHO1=0.
      RHO2=DELR
      PX1=(X(1)-XMIN)/XINC
      PY1=(Y(1)-YMIN)/YINC
      UP1=UP(1)
      IF(UP1) GO TO 10
C
C  GO TO FIRST POSITION WITH PEN UP
      CALL PLOT(PX1,PY1,3)
C
   10 DO 40 I=2,NRPTS
      PX2=(X(I)-XMIN)/XINC
      PY2=(Y(I)-YMIN)/YINC
      UP2=UP(I)
      IF(UP2) GO TO 22
      IF(UP1) GO TO 37
      IF(KK .EQ. 2) GO TO 38
      DELX=PX2-PX1
      DELY=PY2-PY1
      RHO=SQRT(DELX**2+DELY**2)
      RHO1=RHO1+RHO
      IF(RHO2 .GT. RHO1) GO TO 38
      DELX=DELX*DELR/RHO
```

```
      DELY=DELY+DELR/RHO
      DX 6=DELX*.1
      DY 6=DELY*.1
      IF(DR .EQ. 0.) GO TO 20
      DX=DELX*DR/DELR
      DY=DELY*DR/DELR
      PX1=PX1+DX
      PY1=PY1+DY
      GO TO 21
20    IF(RHO2 .GT. RHO1) GO TO 38
      PX1=PX1+DELX
      PY1=PY1+DELY
21    CALL PLOT(PX1,PY1,IP)
      IF(KK .EQ. 6) CALL PLOT(PX1+DXG,PY1+DY6,2)
      J=J+1
      IP=IPEN(J0+MOD(J,JC))
      RHO2=RHO2+DELR
      GO TO 20
22    DR=0.
      RHO1=0.
      RHO2=DELR
      GO TO 39
C PEN HAS BEEN UP, PREPARE TO LOWER PEN
37    CALL PLOT(PX2,PY2,3)
      GO TO 39
38    CALL PLOT(PX2,PY2,IP)
39    DR=RHO2-RHO1
      PX1=PX2
      PY1=PY2
      UP1=UP2
40    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CLIN EQ (A, B, X, N, N DIM, IFLAG, ERR)

C     CLIN EQ USES L-U DECOMPOSITION TO
C     FIND THE TRIANGULAR MATRICES L, U
C     SUCH THAT  L * U = A.  L AND U ARE
C     STORED IN A.  THIS FORM IS USED WITH
C     BACK-SUBSTITUTION TO FIND THE SOLN
C     X OF  A * X = L * U * X = B.
C     N IS THE NUMBER OF EQUATIONS AND
C     N DIM IS THE DIMENSION OF ALL ARRAYS
C     IN THE PARAMETER LIST.
C
C     IF IFLAG = 0, L, U, AND X ARE
C     COMPUTED.
C     IF IFLAG IS NON-ZERO, IT IS ASSUMED
C     THAT L AND U HAVE BEEN COMPUTED IN
C     A PREVIOUS CALL AND ARE STILL STORED
C     IN A.  THUS ONLY X IS COMPUTED.
C     ERR IS THE ESTIMATED RELATIVE
C     ERROR OF THE SOLUTION VECTOR.
C
      COMPLEX*16   A, B, X, T
      REAL*8 ERR
      DIMENSION  A(N DIM, N DIM), B(N DIM), X(N DIM)
      DIMENSION  IROW(51), Q(51)
      DATA  EPS /1.0E-15/
C
      IF (N.GT.51)  GO TO 900
      IF(N .EQ. 1)  GO TO 850
      IF (IFLAG.NE.0)  GO TO 600
      DO 050 I = 1,N
      Q(I) = 0.0
      DO 040 J = 1,N
      QQ = CDABS (A(I,J))
040   IF (Q(I).LT.QQ)  Q(I) = QQ
      IF (Q(I).EQ.0.0)  GO TO 901
050   CONTINUE
      ERR = EPS
      PPIV = 0.0
      DO 100 I = 1,N
100   IROW(I) = I
C
      DO 500  L = 1,N
      PIVOT = 0.0
      K = L - 1
      DO 240  I = L,N
      IF (K.LT.1)  GO TO 230
      DO 220 J = 1,K
220   A(I,L) = A(I,L) - A(J,L) * A(I,J)
230   F = CDABS (A(I,L)) / Q(I)
      IF (PIVOT.GT.F)  GO TO 240
      PIVOT = F
      NPIVOT = I
240   CONTINUE
      IF (PIVOT.EQ.0.0)  GO TO 901
      IF (PPIV.LE.PIVOT)  GO TO 250
```

91

```
57          ERR = ERR * PPIV / PIVOT
58          IF (ERR.GE.1.0D0)  GO TO 901
59   250    PPIV = PIVOT
60          IF (NPIVOT.EQ.L)  GO TO 280
61          Q(NPIVOT) = Q(L)
62          J = IROW(L)
63          IROW(L) = IROW(NPIVOT)
64          IROW(NPIVOT) = J
65          DO 260  I = 1,N
66          T = A(L,I)
67          A(L,I) = A(NPIVOT,I)
68          A(NPIVOT,I) = T
69   260    CONTINUE
70   280    IF (L.EQ.N)  GO TO 500
71          T = (1.0D0,0.0D0) / A(L,L)
72          K = L + 1
73          M = L - 1
74          DO 450  I = K,N
75          IF (M.LT.1)  GO TO 400
76          DO 350  J = 1,M
77   350    A(L,I) = A(L,I) - A(L,J) * A(J,I)
78   400    A(L,I) = T * A(L,I)
79   450    CONTINUE
80   500    CONTINUE
81          IF (ERR.GT.1.0D-5).  PRINT 998, ERR
82   C
83   600    DO 620  I = 2,N
84   620    X(I) = (0.0D0,0.0D0)
85          J = IROW(1)
86          X(1) = B(J) / A(1,1)
87          DO 700  I = 2,N
88          J = IROW(I)
89          K = I - 1
90          DO 650  L = 1,K
91   650    X(I) = X(I) + A(I,L) * X(L).
92          X(I) = (B(J) - X(I)) / A(I,I)
93   700    CONTINUE
94          K = N - 1
95          DO 800  I = 1,K
96          J = N - I
97          M = J + 1
98          DO 800  L = M,N
99   800    X(J) = X(J) - X(L) * A(J,L)
100  800    CONTINUE
101         RETURN
102  C
103  850    X(1)=B(1)/A(1,1)
104         RETURN
105  C
106  900    PRINT 999
107         ERR = 1.0D0
108         RETURN
109  901    PRINT 997
110         ERR = 1.0D0
111         RETURN
112  997    FORMAT ('OERROR IN CLIN EQ, MATRIX IS SINGULAR')
113  998    FORMAT ('OCAUTION-',
```

92

```
114   $ ' CLIN EQ HAS DECOMPOSED AN ILL-CONDITIONED MATRIX.',/,
115   $ ' RESULTS WILL HAVE RELATIVE ERROR =',E11.2)
116   999 FORMAT (' 0ERROR IN CLIN EQ, MATRIX SIZE GREATER THAN 51')
117   END
```

```fortran
      SUBROUTINE HTGAIN(IOPT,FREQ,SIGMA,EPSR,ALPHA,NRMODE,TP,Z,HG)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMPLEX*16 TP(1),HG(3,1),HG0/(0.D0,1.45749544D0)/,
     $          S,C,SSQ,CSQ,NGSQ,SQROOT,RATIO,A1,A2,A3,A4,
     $          P0,H10,H20,H1PRMO,H2PRMO,PZ,H1Z,H2Z,H1PRMZ,H2PRMZ,EXPZ,
     $          1/(0.D0,1.D0)/,MI/(0.D0,-1.D0)/,ONE/(1.D0,0.D0)/
      REAL*8 K,KA13,KA23
      DATA DTR/1.745329252D-02/
C
      OMEGA=6.28318530720D3*FREQ
      NGSQ=DCMPLX(EPSR,-SIGMA/(8.85434D-12*OMEGA))
      K=2.0958426D-02*FREQ
      IF(ALPHA .EQ. 0.D0) GO TO 5
      AK=ALPHA/K
      AK13=CEXP(DLOG(AK)/3.D0)
      AK23=AK13**2
      KA13=1.D0/AK13
      KA23=KA13**2
      P1=KA23*ALPHA*Z
      EXPZ=DEXP(.5D0*ALPHA*Z)
    5 S=COSIN(TP(M)*DTR)
      DO 20 M=1,NRMODE
      SSQ=S*S
      CSQ=ONE-SSQ
      SQROOT=CDSQRT(NGSQ-SSQ)
      TEST=TP(M)*I
      IF(TEST .GT. 10.D0 .OR. ALPHA .EQ. 0.D0) GO TO 10
      P0=KA23*CSQ
      PZ=P0+P1
      CALL MDHNKL(P0,H10,H20,H1PRMO,H2PRMO,TP(M),'HG 1')
      CALL MDHNKL(PZ,H1Z,H2Z,H1PRMZ,H2PRMZ,TP(M),'HG 2')
      A1=H10    +H2Z     +H20
      A2=H1PRMO*H2Z     -H1Z     +H2PRMO
      A3=H10    +H2PRMZ-H1PRMZ*H20
      A4=H1PRMO*H2PRMZ-H1PRMZ*H2PRMO
      RATIO=SQROOT/NGSQ
      C=.5D0*AK23+KA13*MI*RATIO
      HG(1,M)=EXPZ*(C*A1+A2)
      HG(2,M)=KA13*MI*SQROOT*A1+A2
      HG(3,M)=.5D0*AK*MI*HG(1,M)+AK13*MI*EXPZ*(C*A3+A4)
      IF(IOPT .EQ. 1) GO TO 20
      HG(1,M)=HG(1,M)/HG0
      HG(2,M)=HG(2,M)/HG0
      HG(3,M)=HG(3,M)/(RATIO*HG0)
      GO TO 20
   10 C=CDSQRT(CSQ)
      EXPZ=CDEXP(DCMPLX(0.D0,K*Z)*C)
      A1=(NGSQ*C-SQROOT)/(NGSQ*C+SQROOT)
      A2=(C-SQROOT)/(C+SQROOT)
      HG(1,M)=EXPZ+A1/EXPZ
      HG(2,M)=EXPZ+A2/EXPZ
      HG(3,M)=(EXPZ-A1/EXPZ)*C
      IF(IOPT .EQ. 1) GO TO 20
      HG(1,M)=HG(1,M)/(ONE+A1)
      HG(2,M)=HG(2,M)/(ONE+A2)
      HG(3,M)=HG(3,M)/((ONE-A1)*C)
```

94

```
20    CONTINUE
      RETURN
      END
```

57
58
59

```
      SUBROUTINE MDHNKL (Z,M1,M2,H1PRME,H2PRME,THETA,IDBG)
      IMPLICIT COMPLEX*16 (A-H,O-Z)
      COMPLEX*16 1,M1,MPOWER,MTERM
      REAL*8 A,B,C,D,CAP,PART1,PART2,ZMAG
      DIMENSION A(30), B(30), C(30), D(30), CAP(30), PART1(2), PART2(2)
      EQUIVALENCE (PART1,TERM4), (PART2,SUM4)
C
      DATA A  /
     $ 9.30436716929229440190D-01,    3.10145572309743149IID+01,
     $ 2.06763714873162090097D+02,    5.74343652425450274490D+02,
     $ 8.70217655190076172340D+02,    8.20778719228643973200D+02,
     $ 5.41685437404342465420D+02,    2.57945446303020221IID+02,
     $ 9.34504950663116742310D+01,    2.66263518707440066620D+01,
     $ 6.12100043005610727940D+00,    1.15928038448032334720D+00,
     $ 1.04012759441321116G16D-01,    2.48330309637410480030D-02,
     $ 2.08420800972602183000D-03,    2.91334142396567861380D-04,
     $ 2.58274948931275364GD-05,      2.02560587398531400630D-06,
     $ 1.41557353660748070734D-07,    8.86950900130004431240D-09,
     $ 5.01022034632793U09D-10,       2.56580749341156055260D-11,
     $ 1.19610064960912UGGGD-12,      5.09U80924012072031U50D-14,
     $ 1.99403929095177163880D-15,    7.1U8610086312690579700D-17,
     $ 2.39360955255167851120D-18,    7.38030108812246452550D-20,
     $ 2.11942085514075287620D-21,    5.66538586324713410930D-23/
      DATA B  /
     $ 6.78290725144275800456D-01,    1.13049787524045980330D+01,
     $ 5.30323215430760097040D+01,    1.19629040787350243760D+02,
     $ 1.53371031778654150410D+02,    1.27809193148878460900D+02,
     $ 7.47422182157104006310D+01,    3.23559386215231170600D+01,
     $ 1.07855120873041039060D+01,    2.05325737403202090005D+00,
     $ 6.13603736350972235950D-01,    1.09376700098212519660D-01,
     $ 1.64229399546065644650D-02,    2.10550512239571339110D-03,
     $ 2.33167787640721305710D-04,    2.25282A00G60939255661D-05,
     $ 1.91567080450163749595D-06,    1.44469094758796188390D-07,
     $ 9.72861244160977697300D-09,    5.08542797439187958910D-10,
     $ 3.21600060323431464440D-11,    1.59527020452551163510D-12,
     $ 7.21518862291050037780D-14,    2.98765574447639767170D-15,
     $ 1.13605530611735071040D-16,    3.98096598637666916030D-18,
     $ 1.29469847009953559130D-19,    3.09851993405460082200D-21,
     $ 1.09202223904014070360D-22,    2.05272306815955795812D-24/
      DATA C  /
     $ 4.65218350461614724100D-01,    6.20291446194U06298227D+00,
     $ 2.58454643591452623020D+01,    5.22130593114045703920D+01,
     $ 6.21584039421402900120D+01,    4.07516U93663908218970D+01,
     $ 2.70842718702171232280D+01,    1.21501940795740090909D+01,
     $ 3.59455750250449002200D+00,    9.18150064508416091470D-01,
     $ 1.91281263432253351990D-01,    3.31222966999437809740D-02,
     $ 4.84241037929504434440D-03,    6.05603G6820424G4583210D-04,
     $ 6.55501820392277685830D-05,    6.19U59H774395000086120D-06,
     $ 5.16549097862550711190D-07,    3.02204U81U0B402150098G0D-08,
     $ 2.52781006537051262770D-09,    1.50330G6103B983B0141D-10,
     $ 8.08229360424G44091570D-12,    3.94739614377010544710D-13,
     $ 1.75908919060105126750D-14,    7.181421476226377B9920D-16,
     $ 2.69572878236725B9G41D-17,     9.33585725495154618650D-19,
     $ 2.99226194060959813150D-20,    8.90156757605116207010D-22,
     $ 2.46444285051250333750D-23,    6.36560209353610574090D-25/
      DATA D  /
     $ 6.78290725142758B456D-01,      4.52199150096183921310D+01,
     $ 3.76832625080153267760D+02,    1.19629404787350243440D+03,
     $ 1.99382341312504054B0D+03,     2.04494709038205543750D+03,
     $ 1.42010214609864960900D+03,    7.11830649673500857463D+02,
```

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     $       2.6963282184G02597192D+02,    7.9891206472896585111D+01,
     $       1.9021715826H00139294D+01,    3.7188105233392256682D+00,
     $       6.0764877832340288572D-01,    8.4220204895828535644D-02,
     $       1.0026214868551016149D-02,    1.0363012784032050021D-03,
     $       9.3867869420580235442D-05,    7.5124345274574017960D-06,
     $       5.3507368429103773360D-07,    3.41354U225147290163UD-08,
     $       1.9610093247972931935D-09,    1.02097U0508963274472D-10,
     $       4.8341763773500352579D-12,    2.0913590211334783723D-13,
     $       8.2990437346566602039D-15,    3.0316i41149646260564D-16,
     $       1.0228111791370633117GD-17,   3.1967U6345924779236GD-19,
     $       9.2021903191776400453D-21,    2.5103962999804300309D-22/
C
      DATA CAP /
     $       1.0416666666G6666666G3D-01,   8.3550347222222221160D-02,
     $       1.2822657455G327160I9D-01,    2.9184902646414046315D-01,
     $       8.8162726744375764074D-01,    3.32140H281862767526400+00,
     $       1.4995762986862554546D+01,    7.0923013011586517530D+01,
     $       4.7445153886U2643I807D+02,    3.2074900908906G190040D+03,
     $       2.4006549640U74004605D+04,    1.989231191695097912ID+05,
     $       1.7919020077753438063D+06,    1.7484377180034121023D+07,
     $       1.8370737796763307297UD+08,   2.0679040329451551566HD+09,
     $       2.4827519375935880472D+10,    3.16694549017346873315D+11,
     $       4.2771126865134715582D+12,    6.0971132411392560749D+13,
     $       9.1486942234356396792D+14,    1.4413525170009350101D+16,
     $       2.378U844395175704853993D+17, 4.10460U1600946921885D+18,
     $       7.3900049415704853993D+19,    1.3859220004603943141D+21,
     $       2.7030825930275761623D+22,    5.4747478619645573335D+23,
     $       1.149893701438633524D+25,     2.5014180692753603969D+26/
C
      DATA 1/(0.D0,1.D0)/,MI/(0.D0,-1.D0)/
      DATA ONE/(1.D0,0.D0)/,TWO/(2.D0,0.D0)/,ZERO/(0.D0,0.D0)/
      DATA ROOT3/(1.7320508075G88DD0,0.D0)/
      DATA ALPHA/(8.536672188309510D-1,0.D0)/
      DATA CONST1/( 2.5881904510252D-01,-9.6592582628906D-01)/
      DATA CONST2/( 2.5881904510252D-01, 9.6592582628906D-01)/
      DATA CONST3/(-9.6592582628906D-01, 2.5881904510252D-01)/
      DATA CONST4/(-9.6592582628906D-01,-2.5881904510252D-01)/
C
      ZPOWER=ONE
      SUM3=ZERO
      SUM4=ZERO
      ZMAG=CDABS(Z)
      IF(ZMAG .GT. 6.1D0) GO TO 70
      SUM1=ZERO
      SUM2=ZERO
      ZTERM=-Z**3/(200.D0,0.D0)
      DO 50 M=1,30
      SUM1=SUM1+DCMPLX(A(M),0.D0)*ZPOWER
      SUM2=SUM2+DCMPLX(B(M),0.D0)*ZPOWER
      SUM3=SUM3+DCMPLX(C(M),0.D0)*ZPOWER
      TERM4=DCMPLX(D(M),0.D0)*ZPOWER
      SUM4=SUM4+TERM4
      IF(DABS(PART1(1)/PART2(1)) .LE. 1.D-17 .AND.
     $   DABS(PART1(2)/PART2(2)) .LE. 1.D-17) GO TO 60
      ZPOWER=ZPOWER*ZTERM
   50 CONTINUE
   60 GM2F=I*(Z*SUM2-TWO*SUM1)/ROOT3
      GMFP=I*(SUM4+TWO*Z*Z*SUM3)/ROOT3
      M1=Z*SUM2+GM2F
```

```
114        H2=H1-TWO*GM2F
115        H1PRME=SUM4+GPMFP
116        H2PRME=H1PRME-TWO*GPMFP
117        GO TO 999
118    C
119    70  MPOWER=ONE
120        SUM1=ONE
121        SUM2=ONE
122        RTZ=CDSQRT(Z)
123        SQRTZB=RTZ*Z
124        ZTERM=1/SQRTZB
125        MTERM=-ZTERM
126        DM=ZERO
127        TERM3=ONE
128        DO 80 M=1,30
129        ZPOWER=ZPOWER*ZTERM
130        MPOWER=MPOWER*MTERM
131        DM=DM+ONE
132        TERM1=DCMPLX(CAP(M),0.D0)*ZPOWLR
133        TERM2=DCMPLX(CAP(M),0.D0)*MPOWER
134        IF(CDABS(TERM2/TERM3) .GE. 1.D0) GO TO 81
135        SUM1=SUM1+TERM1
136        SUM2=SUM2+TERM2
137        SUM3=SUM3+DM*TERM1
138        TERM4=DM*TERM2
139        SUM4=SUM4+TERM4
140        IF(DABS(PART1(1)/PART2(1)) .LE. 1.D-17 .AND.
141      $  DABS(PART1(2)/PART2(2)) .LE. 1.D-17) GO TO 81
142        TERM3=TERM2
143    80  CONTINUE
144    81  ZTERM=(-1.5D0,0.D0)/Z
145        SUM3=SUM3*ZTERM
146        SUM4=SUM4*ZTERM
147        TERM1=((-0.25D0,0.D0)-1*SQRTZB)/Z
148        TERM2=((-0.25D0,0.D0)+1*SQRTZB)/Z
149        EXP1=CDEXP((0.D0,0.66666666666666667D0)*SQRTZB)
150        EXP2=CONST1*EXP1
151        EXP3=CONST12/EXP1
152        EXP4=CONST3*CXP1
153        EXP5=CONST4/EXP1
154        ZTERM=ALPHA/CDSQRT(RTZ)
155        TERM4=Z
156        IF(PART1(1) .GE. 0.D0 .OR. PART1(2) .GE. 0.D0) GO TO 90
157        H1=ZTERM*(EXP2*SUM2+EXP5*SUM1)
158        H1PRME=ZTERM*(EXP2*(SUM2*TERM4+SUM4)+EXP5*(SUM1*TERM1+SUM3))
159        GO TO 110
160    90  H1=ZTERM*EXP2*SUM2
161        H1PRME=ZTERM*EXP2*(SUM2*TERM4+SUM4)
162    110 IF(PART1(1) .GE. 0.D0 .OR. PART1(2) .LT. 0.D0) GO TO 120
163        H2=ZTERM*(EXP3*SUM1+EXP4*SUM2)
164        H2PRME=ZTERM*(EXP3*(SUM1*TERM1+SUM3)+EXP4*(SUM2*TERM4+SUM4))
165        GO TO 999
166    120 H2=ZTERM*EXP3*SUM1
167        H2PRME=ZTERM*EXP3*(SUM1*TERM1+SUM3)
168    C
169    C   CALCULATE WRONSKIAN AS PARTIAL CHECK ON VALIDITY
170    999 SUM4=H1*H2PRME-H1PRME*H2
```

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```
171       IF(DABS(PART2(1)) .LE. 1.D-8 .AND.
172      $   DABS(PART2(2))+1.4574954104046IDO) .LE. 1.D-8) GO TO 1000
173       PRINT 1001,SUM4,THETA,IDBG
174       RETURN
175 1000  
176 1001  FORMAT(' ***** POSSIBLE ERROR IN MOHNKL: W = ',1P2E15.6,
177      $   ' FOR THETA = ',0P2F10.4,' AT ',A4)
          END
```

99

APPENDIX B:

BUMP

```
      PROGRAM 'BUMP'

C     PROGRAM TO READ SEVERAL RADIALS WITH, SAY, VARYING H'.
C     THIS PROGRAM THEN GENERATES A SERIES OF DATA SETS COMPATIBLE
C     WITH FASTWC INPUT.  THESE DATA SETS CAN BE USED TO GENERATE
C     A SERIES OF MODE SUMS TO SHOW THE EFFECT OF VARYING, SAY, THE
C     IONOSPHERE ALONG A PATH FOR WHICH THE GROUND CONDUCTIVITY OR
C     THE MAGNETIC FIELD VARIES SIGNIFICANTLY.
C
      LOGICAL DEBUG/.FALSE./
      CHARACTER*4 BCD,SAVE,ID
      DIMENSION BCD(20), INDEX(7, 801), SAVE(81,20), HPRIME(7), ID(20)
C
C     RHOO = START DISTANCE OF BUMP
C     RHOL = END   DISTANCE OF BUMP
C     DRHO  = INTERVAL BETWEEN BUMPS
C     DRHO1 = DISTANCE INCREMENT BETWEEN HPRIME DATA SETS
C     DRHO2 = LENGTH OF BOTTOM OF BUMP
C     RHOMAX = TOTAL PATH LENGTH
C
C     HPRIME = IONOSPHERIC HEIGHT PARAMETER FOR EACH DATA SET
C
      NAMELIST/DATUM/DEBUG,RHOO,RHOL,DRHO,DRHO1,DRHO2,RHOMAX,HPRIME
C
      DATA RHOO/1./,RHOL/2./,DRHO/.1/,DRHO1/.1/,DRHO2/20./,IMAX/801/,
     $  RHOMAX/10./,ID/20*'  '/,
     $  HPRIME/B7.,B5.,B3.,81.,79.,77.,7B.,/
C
C...SET UP RANDOM ACCESS FILES
      DEFINE FILE 11 ( 801,1229,U,J1)
      DEFINE FILE 12 ( 801,1229,U,J2)
      DEFINE FILE 13 ( 801,1229,U,J3)
      DEFINE FILE 14 ( 801,1229,U,J4)
      DEFINE FILE 15 ( 801,1229,U,J5)
      DEFINE FILE 16 ( 801,1229,U,J6)
      DEFINE FILE 17 ( 801,1229,U,J7)
C
C...SET UP LOGICAL FILE FOR OUTPUT FILE
      CALL FACSF('OUSE 2,OUTPUT . ')
C
C
    9 MF=10
      MR=0
   10 READ(5,1000,END=900) BCD
      PRINT 1001,BCD
      IF(SUBSTR(BCD(1),1,4) .EQ. 'DATA') GO TO 100
      IF(SUBSTR(BCD(1),1,4) .EQ. 'STAR') GO TO 200
      IF(SUBSTR(BCD(1),1,4) .EQ. 'NAME') GO TO 20
      IF(SUBSTR(BCD(1),1,4) .EQ. 'ID  ') GO TO 30
      GO TO 10
C
C...READ NAMELIST DATA
   20 READ DATUM
      DRHO=SIGN(DRHO,RHOL-RHOO)
      PRINT DATUM
      GO TO 9
C
```

```
C...ID FOR OUTPUT FILE
30    ID(1)='BUMP'
      READ 1000,(ID(I),I=2,19)
      GO TO 10
C
C...READ INPUT DATA FOR HPRIME(MR)
C   DATA IS TO BE STORED IN RANDOM ACCESS FILE MF
100   MF=MF+1
      MR=MR+1
      IF(MR .GT. 7) GO TO 800
      7=HPRIME(MR)
      READ 1000,BCD
      PRINT 1001,BCD
      NR=0
      I1=1
      I2=0
101   READ 1100,R,F,A,C,B,S,E
      IF(R .EQ. 40.) GO TO 110
      IF(NR .EQ. 0) GO TO 103
C
C...CALCULATE RECORD NUMBER FOR RANDOM ACCESS FILE
C   BASED ON 5 KM RESOLUTION
      I2=R*20.+0.99
      DO 102 I=I1,I2
102   INDEX(MR,I)=NR
103   NR=NR+1
      FI=I2
      PRINT 1100,R,F,A,C,B,S,E,FI
      NM=0
104   NM=NM+1
C...DATA FOR EACH INPUT DISTANCE IS READ AND STORED AS LITERAL DATA
C   A MAXIMUM OF 30 MODES MAY BE INPUT
      IF(NM .GT. 61) GO TO 801
      READ 1000,(SAVE(NM,I),I=1,20)
      IF(SUBSTR(SAVE(NM,I),1,4) .EQ. '    ') GO TO 105
      GO TO 104
105   WRITE(MF'NR) R,F,A,C,B,S,E,T,NM,SAVE
      I1=I2+1
      GO TO 101
110   DO 111 I=11,IMAX
111   INDEX(MR,I)=NR
      LASTM=MR
      GO TO 10
C
C...BEGIN
200   RHO1=RHO0
C...CHECK FOR 1D CARD
      IF(SUBSTR(ID(1),1,4) .EQ. '    ') GO TO 201
      WRITE(2,1000) ID
C...INITIALIZE FILE AND RECORD INDICES
201   MF=11
      MR=1
      INC=1
      RLAST=0.
      RHO=RHO1
      IF(DEBUG) PRINT 1002,   RHO,DRHO1,DRHO2,(HPRIME(I),I=1,LASTM)
      WRITE(2,1200) RHO,DRHO1,DRHO2,(HPRIME(I),I=1,LASTM)
```

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114          I1=1
115    C...DETERMINE RANGE OF RECORDS TO BE USED IN CURRENT STEP
116    202   N1=INDEX(MR,I1)
117          IF(RHO .GT. RHOMAX) RHO=RHOMAX
118          I2=RHO+20.+.99
119          IF(I2 .LT.    1) GO TO 220
120          IF(I2 .GT. IMAX) I2=IMAX
121          N2=INDEX(MR,I2)
122          IF(DEBUG) PRINT 1003,MR,I1,I2,N1,N2
123          DO 210 N=N1,N2
124          READ(MF'N) R,F,A,C,B,S,E,T,NM,SAVE
125          IF(N .GT. N1) GO TO 207
126          R1=R
127    207   IF(R .GE. RLAST) GO TO 208
128    C...ADJUST R VALUES TO CORRESPOND TO BEGINNING OF CURRENT STEP
129    C   USE DATA FROM PREVIOUS INPUT R VALUE
130    C   EXAMPLE:  SUPPOSE DATA WAS INPUT AT RHO=1.0 AND 1.2 MM.
131    C             IF AN R VALUE GT 1.0 AND LT 1.2 MM IS REQUIRED, THEN THE
132    C             PROGRAM WILL USE THE DATA FROM 1.0 MM AT THIS R VALUE.
133    C
134    C
135          DR=R-R1
136          R1=R
137          R=RLAST+DR
138          GO TO 209
139    208   R1=R
140    209   IF(DEBUG) PRINT 1100,R,F,A,C,B,S,E,T
141          WRITE(2,1100) R,F,A,C,B,S,E,T
142    210   WRITE(2,1000) ((SAVE(M,I),I=1,20),M=1,NM)
143          RLAST=RHO
144          IF(RHO .EQ. RHOMAX) GO TO 290
145          (I2 .EQ. IMAX) GO TO 290
146          I1=I2+1
147    220   MF=MF+INC
148          MR=MR+INC
149          IF(MR .EQ. LASTM) GO TO 230
150          IF(MR .GT. LASTM) GO TO 240
151          IF(MR .EQ. 1) GO TO 250
152          IF(MR .LT. 1) GO TO 290
153    221   RHO=RHO+DRHO1
154          GO TO 202
155    C
156    230   RHO=RHO+DRHO2
157          GO TO 202
158    240   MF=MF-2
159          MR=MR-2
160          INC=-1
161          GO TO 221
162    250   RHO=40.
163          GO TO 202
164    C
165    290   WRITE(2,1201)
166          RHO1=RHO1+DRHO
167          IF(DRHO .LT. 0.) GO TO 291
168          IF(RHO1 .GT. RHOL) GO TO 9
169          GO TO 201
170    291   IF(RHO1 .LT. RHOL) GO TO 9
             GO TO 201
```

```
C
800   PRINT 1800
      GO TO 900
801   PRINT 1801
900   STOP
1000  FORMAT(20A4)
1001  FORMAT(1X,20A4)
1002  FORMAT(' ORHO=',F5.2,' DRHO1=',F5.2,' DRHO2=',F5.2,' H''=',7F4.0)
1003  FORMAT(80X,5I5)
1100  FORMAT(1X,F7.3,2X,F8.4,2(2X,F8.3),2(2X,1PE10.3),2(2X,0PF5.1)))
1200  FORMAT('DATA'
     $       'RHO=',F5.2,' DRHO1=',F8.9,' DRHO2=',F5.2,' H''=',7F4.0)
1201  FORMAT(' R 40.')
1800  FORMAT(' OTOO MANY HPRIMES')
1801  FORMAT(' OTOO MANY MODES')
      END
```